Research Summary

Anomalous Diffusion in SRV

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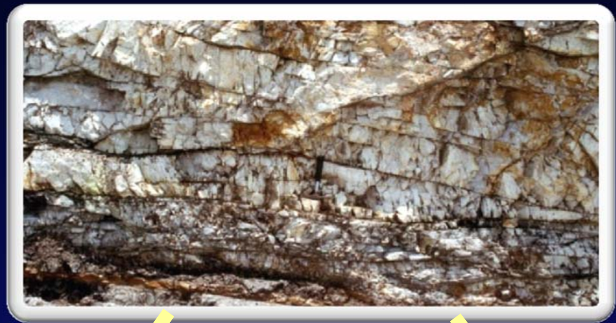
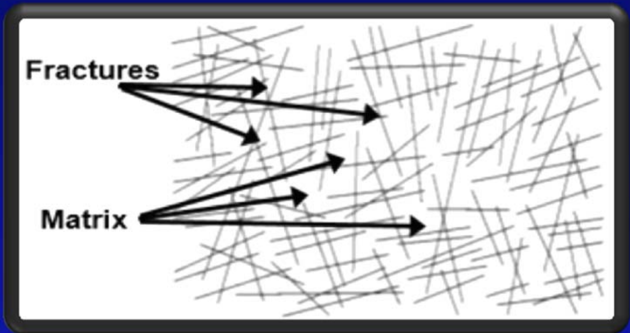
Problem Statement

Complex Network of Fractures



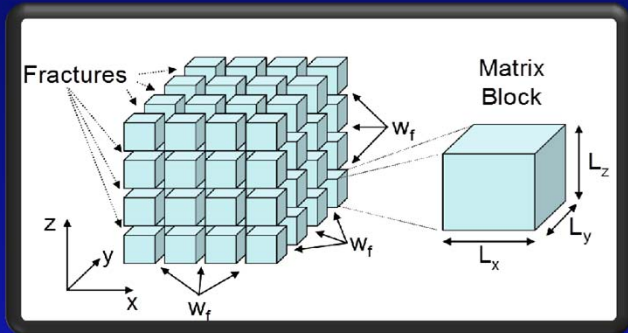
Large variations of
- scale
- connectivity
- conductivity

Discrete Fracture Network Model (DFN)



Not the tool of choice for most routine engineering applications

Dual-Porosity Models



Practical but highly idealized

Problem Statement

Complex Network of Fractures

Alternatives to account for the non-uniform distribution of fractures

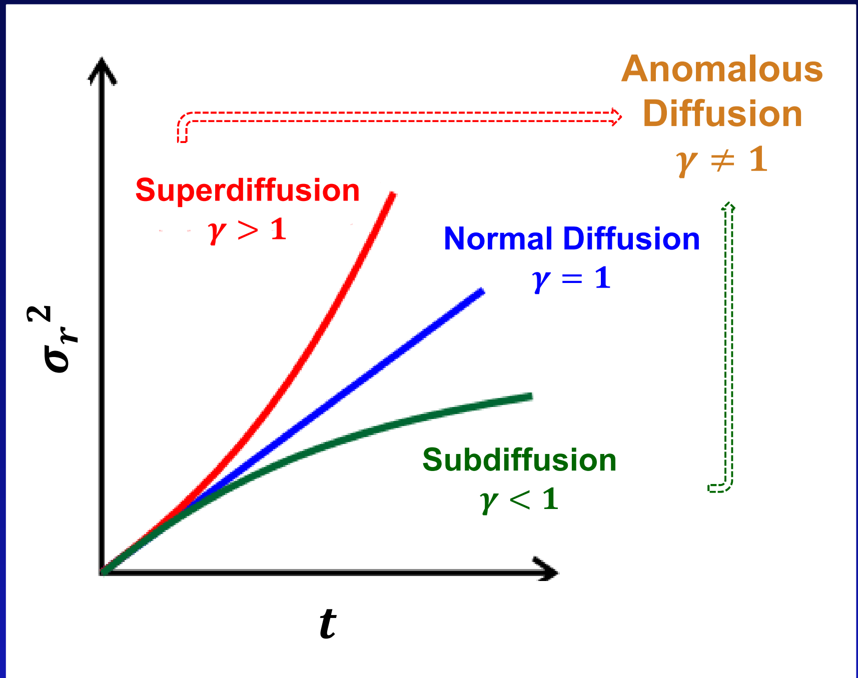
ANOMALOUS (FRACTIONAL) DIFFUSION

Background

Diffusion Result of the Random Brownian Motion of individual particles

General relationship between ${\sigma_r}^2$ and t :

$$\boxed{{\sigma_r}^2 \sim D t^{\gamma}}$$



Scope of Research

To investigate the potential of the <u>ANOMALOUS</u> <u>DIFFUSION</u> concept as an alternative to the dual-porosity idealizations of the SRV.

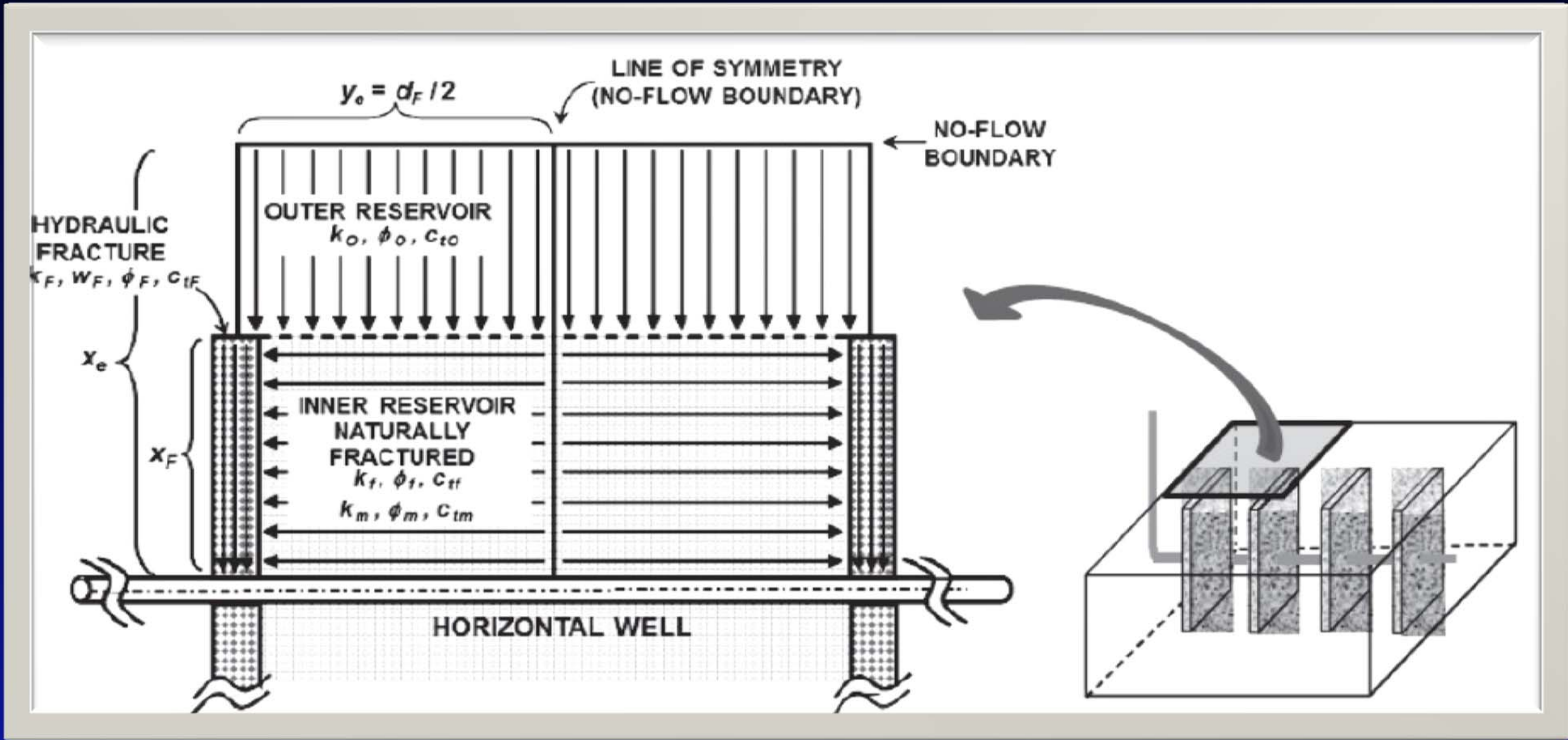
Approach

The TRILINEAR FLOW MODEL is modified

by replacing the dual-porosity idealization

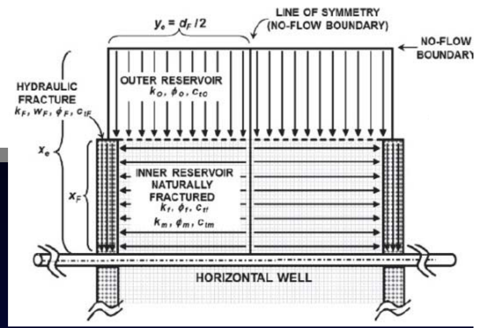
with the anomalous assumption.

Trilinear Flow Model



(Brown, 2009)

Trilinear Flow Model



Trilinear Dual-Porosity Model (TDP)

Inner Res. ⇒ Dual-Porosity

$$v_I = -\frac{k}{\mu}\frac{\partial \Delta p_I}{\partial x}$$

Trilinear Anomalous Diffusion Model (TAD)

Inner Res. ⇒ Anomalous Diff.

$$v_I = -\lambda_\alpha \frac{\partial^{1-\alpha}}{\partial t^{1-\alpha}}\left(\frac{\partial \Delta p_I}{\partial x}\right), \qquad 0 < \alpha < 1$$

$$\lambda_\alpha = \frac{k_\alpha}{\mu}, \quad k_\alpha \equiv \left[L^2 T^{1-\alpha}\right]$$

TDP & TAD Models

TDP

Wellbore Press.

$$\bar{p}_{wD} = (\bar{p}_{FD})_{x_D=0} = \frac{\pi}{sC_{FD}\sqrt{\alpha_F}\tanh(\sqrt{\alpha_F})}$$

$$\alpha_O = \frac{\beta_O}{C_{RD}y_{eD}} + u$$

$$\beta_O = \sqrt{s/\eta_{OD}}\,tanh\left[\sqrt{s/\eta_{OD}}\,(x_{eD}-1)\right]$$

$$\alpha_F = \frac{2\beta_F}{C_{FD}} + \frac{s}{\eta_{FD}}$$

$$\beta_F = \sqrt{\alpha_O}\,tanh[\sqrt{\alpha_O}(y_{eD} - w_D/2)]$$

TAD

$$\bar{p}_{wD} = (\bar{p}_{FD})_{x_D=0} = \frac{\pi}{sC_{FD}\sqrt{\alpha_F}\tanh(\sqrt{\alpha_F})}$$

$$\alpha_O = \left(\frac{x_F^2}{\eta_I}\right)^{1-\alpha} s^{\alpha}\left[1 + \left(\frac{\lambda_O}{\lambda_I}s^{-1}\beta_O\right)\right]$$

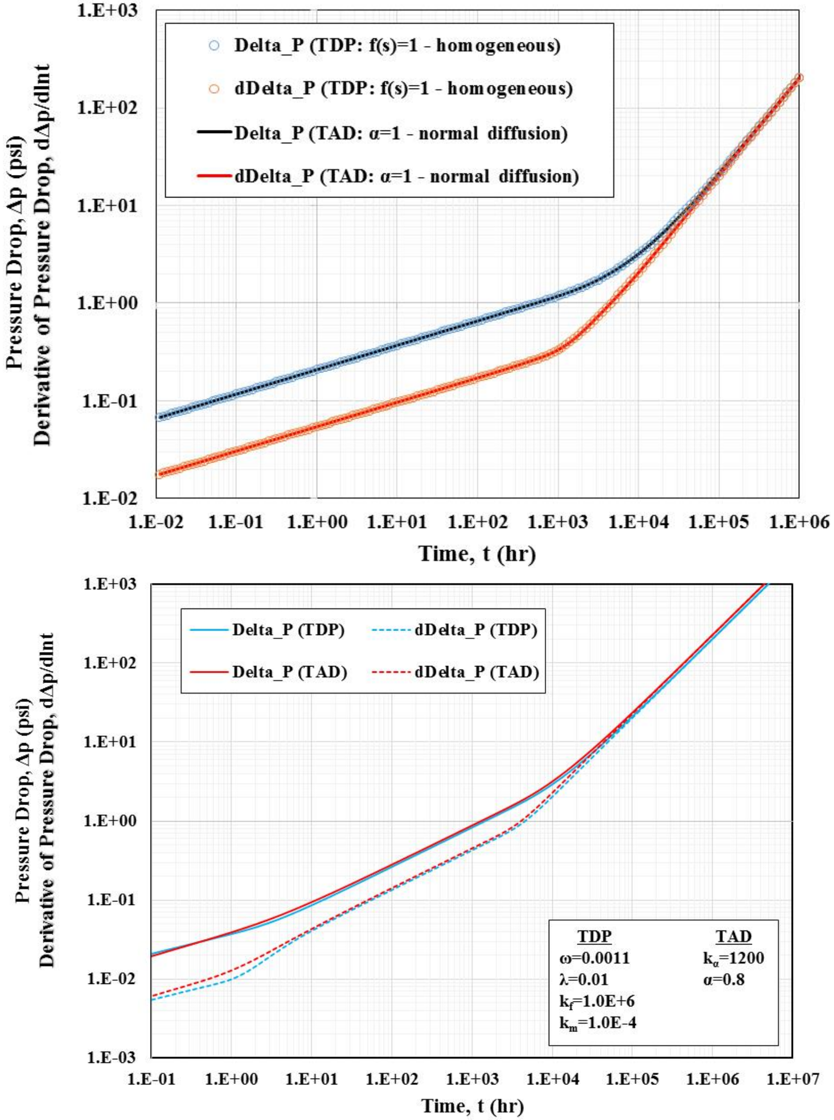
$$\beta_O = \sqrt{s/\eta_{OD}}\,tanh\left[\sqrt{s/\eta_{OD}}\,(x_{eD}-1)\right]$$

$$\alpha_F = \left[\frac{2}{w_D}\frac{\lambda_I}{\lambda_F}\left(\frac{\eta_I}{x_F^2}\right)^{1-\alpha} s^{1-\alpha}\beta_F + \frac{1}{\eta_{FD}}s\right]$$

$$\beta_F = \sqrt{\alpha_O}\,tanh[\sqrt{\alpha_O}(y_{eD} - w_D/2)]$$

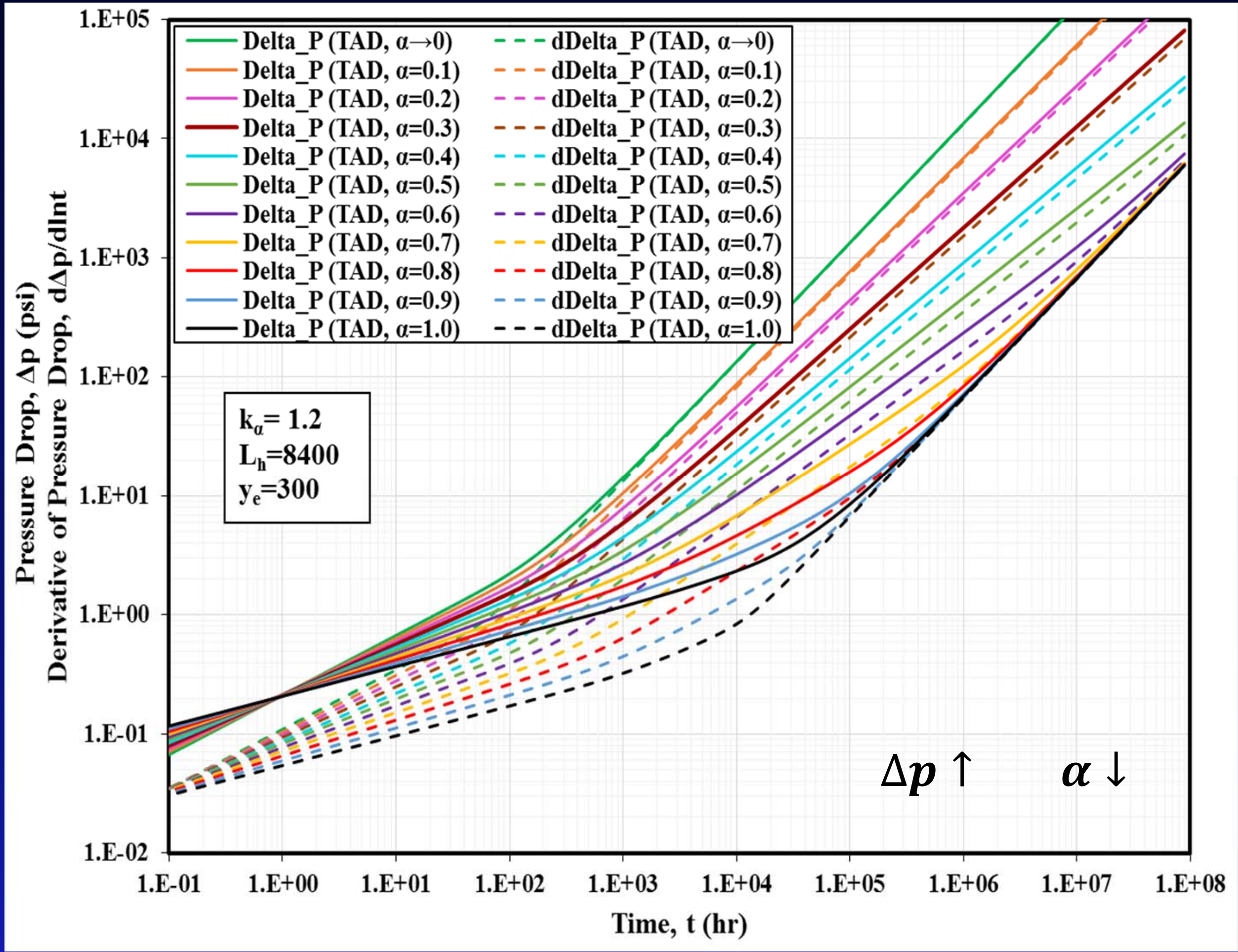
Verification of TAD Model

- TAD Model is verified
  - homogeneous model
  - dual-porosity model
- Asymptotic approximations are developed



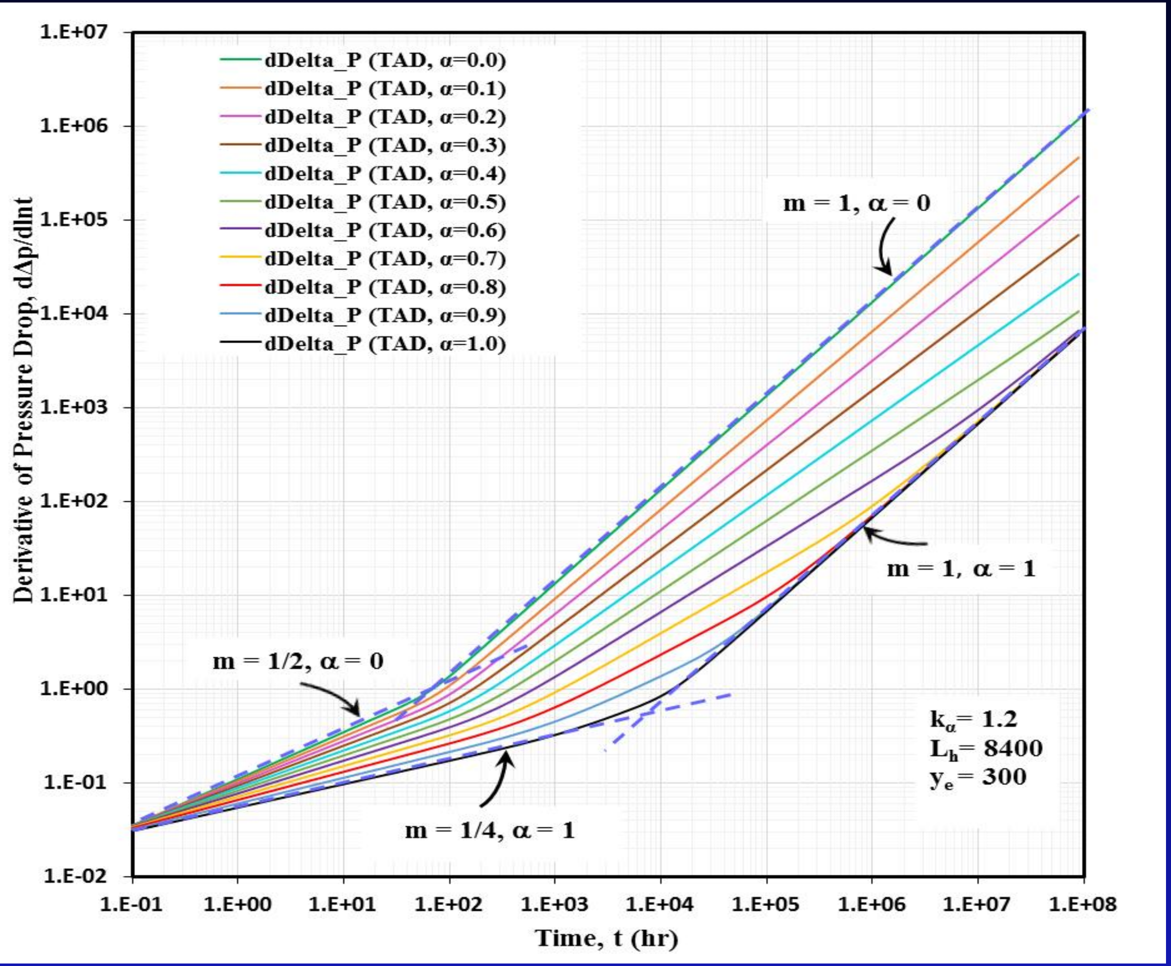
Discussion of Results

Effect of Fractional Order of the Time Derivative (α)



Discussion of Results

Effect of Fractional Order of the Time Derivative (α)



$\alpha \rightarrow 0$

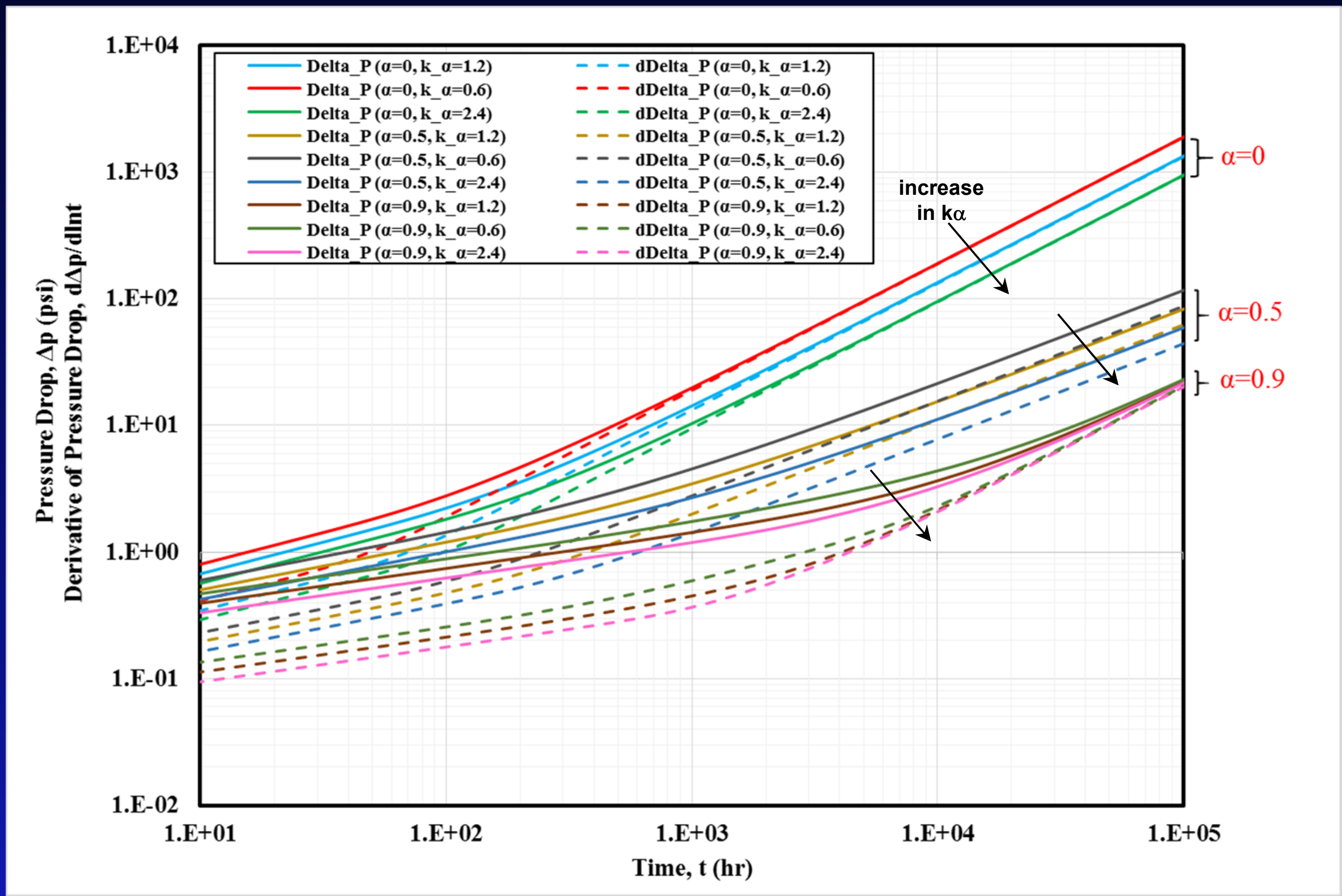
- longer interruptions
- dominance of matrix
- slower depletion

$\alpha \rightarrow 1$

- shorter interruptions
- dominance of fractures
- faster depletion

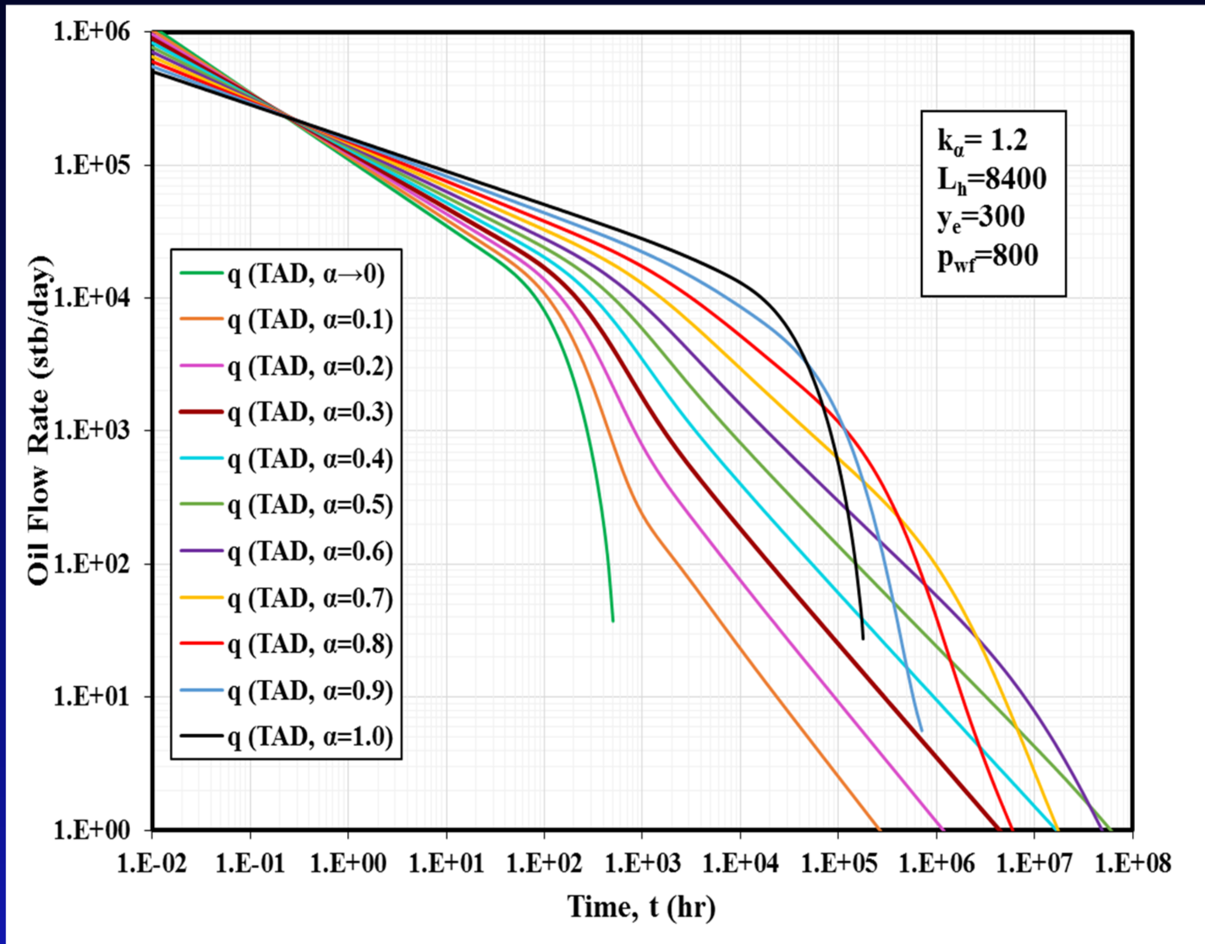
Discussion of Results

Combined Effect of Anomalous Diffusion Parameters (α and k_α)



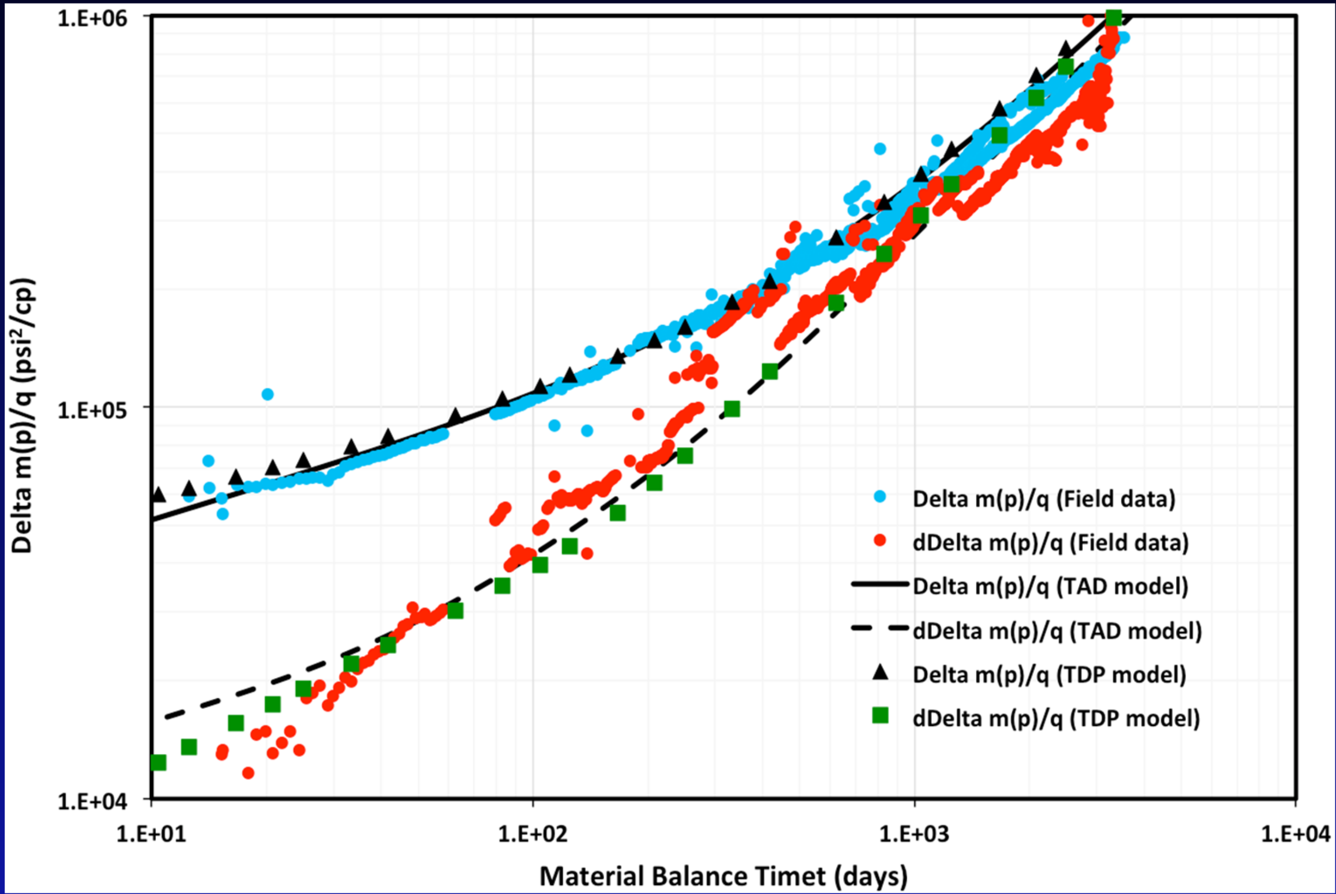
Discussion of Results

Rate Decline Characteristics



Field Application

Barnett Field



Status

Space and Time Fractional Anomalous Diffusion

$$v_I = -\lambda_{\alpha,\beta} \frac{\partial^{1-\alpha}}{\partial t^{1-\alpha}} \left(\frac{\partial^\beta \Delta p_I}{\partial x^\beta} \right), \qquad \lambda_{\alpha,\beta} = \frac{k_{\alpha,\beta}}{\mu}, \qquad 0 < \alpha < 1 \text{ and } 0 < \beta < 1$$

$$\frac{\partial}{\partial x}\left(\lambda_{\alpha,\beta} \frac{\partial^\beta \Delta p_I}{\partial x^\beta} \right) + \frac{\partial}{\partial y}\left(\lambda_{\alpha,\beta} \frac{\partial^\beta \Delta p_I}{\partial y^\beta} \right) = (\phi C_t)_I \frac{\partial^\alpha \Delta p_I}{\partial t^\alpha}$$

$$\boxed{\bar{p}_{ID}(y_D, s) = \bar{p}_{ID}(0, s) E_{\beta+1}\left(\alpha_o y_D^{\beta+1}\right) + \left[\frac{d^\beta}{dy_D^\beta} \bar{p}_{ID}(y_D, s) \right]_{y_D=0} y_D^\beta \, E_{\beta+1,\beta+1}\left(\alpha_o y_D^{\beta+1}\right)}$$

where;

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)}$$

α : *fractional order of the time derivative*

β : *fractional order of the space derivative*

Conclusions

- Anomalous diffusion model can be alternative to dual porosity based models.
- Anomalous diffusion model can capture wide variety of flow behaviors.
- The interpretations of the pressure and flow rate behaviors predicted by the anomalous diffusion model are consistent with the physical expectations and the results of the alternate models.
- Anomalous diffusion model does not require explicit references to the intrinsic properties of the matrix and fracture media.
- TAD model is useful for performance predictions and pressure- and rate-transient analysis of fractured horizontal wells in tight unconventional reservoirs.

Thank You

General Data

WELL, RESERVOIR, AND FLUID DATA (Intrinsic Properties)	
Formation thickness, h, ft	250
Wellbore radius, r_w, ft	0.25
Horizontal well length, L_h, ft	2800
Number of hydraulic fractures, n_F	15
Distance between hydraulic fractures, d_F, ft	200
Distance to boundary parallel to well (1/2 well spacing), x_e, ft	250
Inner reservoir size, y_e, ft	100
Viscosity, μ, cp	0.3
Hydraulic fracture porosity, ϕ_F, fraction	0.38
Hydraulic fracture permeability, k_F, md	5.0E+04
Hydraulic fracture total compressibility, c_{tF}, psi^{-1}	1.0E-04
Hydraulic fracture half-length, x_F, ft	250
Hydraulic fracture width, w_F, ft	0.01
Outer reservoir permeability, k_O, md	1.0E-04
Outer reservoir porosity, ϕ_O	0.05
Outer reservoir compressibility, c_{tO}, psi^{-1}	1.0E-05
Constant flow rate, q, stb/day	150

INNER RESERVOIR DATA			
TDP (Intrinsic Properties)		TAD	
Matrix permeability, k_m, md	1.0E-4	Phenomenological coefficient, k_α, md-hr$^{1-\alpha}$	1.2
Matrix porosity, ϕ_m	0.05	Porosity compressibility product, $(\phi c_t)_\alpha$, psi^{-1}	4.62E-4
Matrix total compressibility, c_{tm}, psi^{-1}	1.0E-5		
Natural fracture permeability, k_f, md	1.0E+3		
Natural fracture porosity, ϕ_f	0.7		
Natural fracture total compressibility, c_{tf}, psi^{-1}	5.5E-1		
Natural fracture width, h_f, ft	3.0E-3		

Barnett Field Data

Formation thickness, h, ft	300
Reservoir temperature, T, R	565.67
Distance to boundary parallel to well (1/2 well spacing), x_e, ft	275
Inner reservoir size, y_e, ft	90.3
Viscosity, μ, cp	0.02
The order of fractional derivative of time, α	0.8
Phenomenological coefficient of anomalous diffusion, k_α, md-hr$^{1-\alpha}$	0.13
Porosity – compressibility product of inner reservoir, $(\phi c_t)_\alpha$,psi^{-1}	2.00E-04
Hydraulic fracture porosity, ϕ_F	0.38
Hydraulic fracture permeability, k_F, md	1.00E+03
Hydraulic fracture total compressibility, c_{tF}, psi^{-1}	1.00E-04
Hydraulic fracture half-length, x_F, ft	275
Hydraulic fracture width, w_F, ft	0.01
Outer reservoir permeability, k_O, md	1.00E-06
Outer reservoir porosity, ϕ_O	0.04
Outer reservoir compressibility, c_{tO}, psi^{-1}	3.00E-04

Constants Used in Asymptotic Approximations

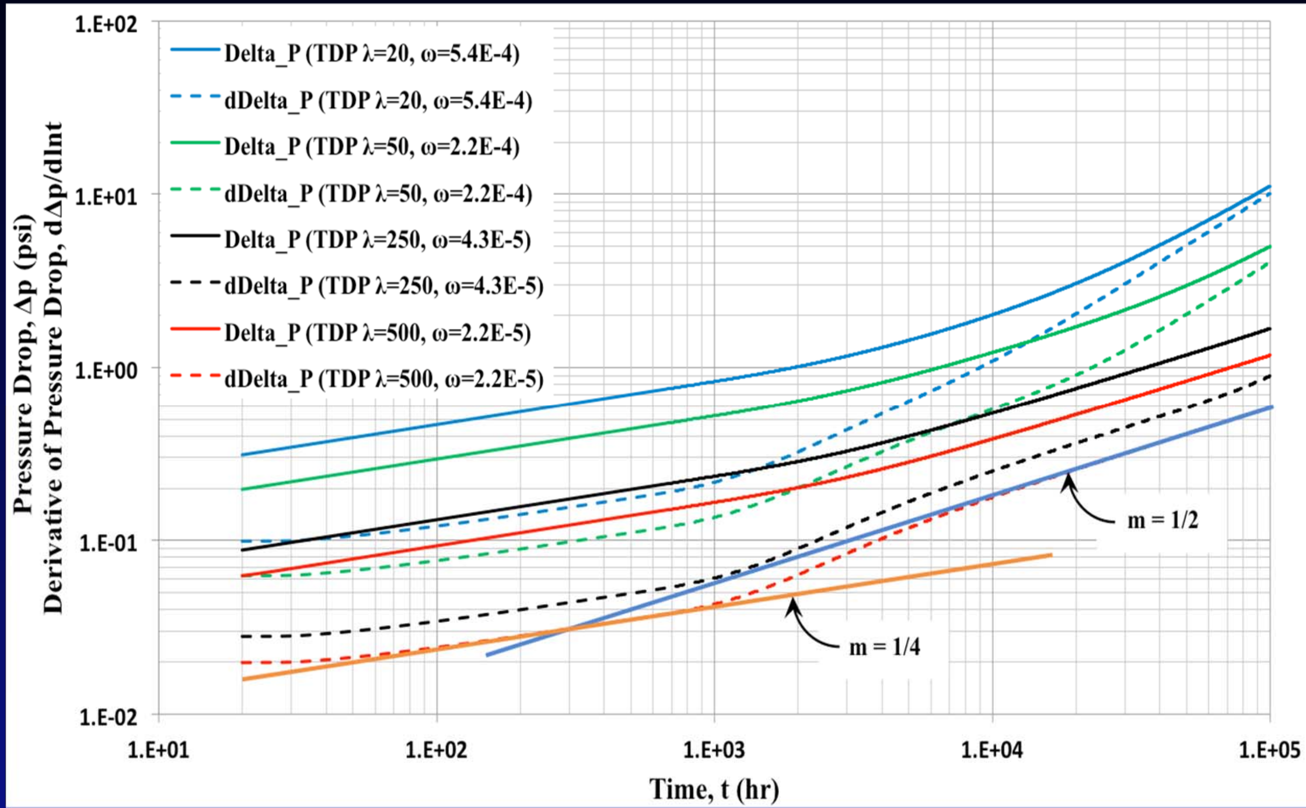
A_{O1}	$\left(\frac{x_F^2}{\eta_\alpha}\right)^{1-\alpha}\left[1+\left(\frac{\lambda_O}{\lambda_\alpha}\right)\left(\frac{x_{eD}-1}{\eta_{OD}}\right)\right]$		
$A_{O2,\alpha}$	$\left(\frac{x_F^2}{\eta_\alpha}\right)^{1-\alpha}\left(\frac{\lambda_O}{\lambda_\alpha}\right)\left(\frac{1}{\sqrt{\eta_{OD}}}\right)$		
$A_{O2,0.5}$	$\left(\frac{x_F^2}{\eta_\alpha}\right)^{0.5}\left(\frac{\lambda_O}{\lambda_\alpha}\right)\left(\frac{1}{\sqrt{\eta_{OD}}}\right)$		
A_{O3}	$\left(\frac{x_F^2}{\eta_\alpha}\right)^{1-\alpha}$		
$A_{F,0}$	$\frac{2}{w_D}\frac{\lambda_\alpha}{\lambda_F}\left(\frac{\eta_\alpha}{x_F^2}\right)B_{F,0}+\frac{1}{\eta_{FD}}$	$B_{F,0}$	$\sqrt{A_{O1}}\tanh\left[\sqrt{A_{O1}}(y_{eD}-w_D/2)\right]$
$A_{F1,\alpha}$	$\frac{2}{w_D}\frac{\lambda_\alpha}{\lambda_F}\left(\frac{\eta_\alpha}{x_F^2}\right)^{1-\alpha}B_{F1,\alpha}$	$B_{F1,\alpha}$	$A_{O1}(y_{eD}-w_D/2)$
$A_{F2,\alpha}$	$\frac{2}{w_D}\frac{\lambda_\alpha}{\lambda_F}\left(\frac{\eta_\alpha}{x_F^2}\right)^{1-\alpha}B_{F2,\alpha}$	$B_{F2,\alpha}$	$\sqrt{A_{O1}}$
$A_{F2,1,\alpha}$	$\frac{2}{w_D}\frac{\lambda_\alpha}{\lambda_F}\left(\frac{\eta_\alpha}{x_F^2}\right)^{1-\alpha}B_{F2,1,\alpha}$	$B_{F2,1,\alpha}$	$A_{O2}(y_{eD}-w_D/2)$
$A_{F2,2,\alpha}$	$\frac{2}{w_D}\frac{\lambda_\alpha}{\lambda_F}\left(\frac{\eta_\alpha}{x_F^2}\right)^{1-\alpha}B_{F2,2,\alpha}$	$B_{F2,2,\alpha}$	$\sqrt{A_{O2,\alpha}}$
$A_{F2,0.5}$	$\frac{2}{w_D}\frac{\lambda_\alpha}{\lambda_F}\left(\frac{\eta_\alpha}{x_F^2}\right)^{1/2}B_{F2,0.5}$	$B_{F2,0.5}$	$\sqrt{A_{O2,0.5}}\tanh\left[\sqrt{A_{O2.05}}(y_{eD}-w_D/2)\right]$
$A_{F3,0}$	$\frac{2}{w_D}\frac{\lambda_\alpha}{\lambda_F}\left(\frac{\eta_\alpha}{x_F^2}\right)B_{F3,0}+\frac{1}{\eta_{FD}}$	$B_{F3,0}$	$\sqrt{A_{O3}}\tanh\left[\sqrt{A_{O3}}(y_{eD}-w_D/2)\right]$
$A_{F3,1,\alpha}$	$\frac{2}{w_D}\frac{\lambda_\alpha}{\lambda_F}\left(\frac{\eta_\alpha}{x_F^2}\right)^{1-\alpha}B_{F3,1,\alpha}+\frac{1}{\eta_{FD}}$	$B_{F3,1,\alpha}$	$A_{O3}(y_{eD}-w_D/2)$
$A_{F3,2,\alpha}$	$\frac{2}{w_D}\frac{\lambda_\alpha}{\lambda_F}\left(\frac{\eta_\alpha}{x_F^2}\right)^{1-\alpha}B_{F3,2,\alpha}$	$B_{F3,2,\alpha}$	$\sqrt{A_{O3}}$

Asymptotic Approximations

Time Range	Conditions	Pressure	Log-log Slope
Late Time	$\alpha \neq 0,\ x_{eD} \neq 1$	$\lim_{\substack{t_D \to \infty \\ \alpha \neq 0}} p_{wD} = \frac{\pi t_D}{C_{FD} A_{F1,\alpha}}$	1
	$\alpha = 0,\ x_{eD} \neq 1$	$\lim_{\substack{t_D \to \infty \\ \alpha = 0}} p_{wD} = \frac{\pi t_D}{C_{FD} A_{F,0}}$	
	$\alpha \neq 0,\ x_{eD} = 1$	$\lim_{\substack{t_D \to \infty \\ \alpha \neq 0}} p_{wD} = \frac{\pi t_D}{C_{FD} A_{F3,1,\alpha}}$	
	$\alpha = 0,\ x_{eD} = 1$	$\lim_{\substack{t_D \to \infty \\ \alpha = 0}} p_{wD} = \frac{\pi t_D}{C_{FD} A_{F3,0}}$	
Late-Intermediate Time	$\alpha \neq 0,\ x_{eD} \neq 1$	$\lim_{\substack{t_D \to \infty \\ \alpha \neq 0}} p_{wD} = \frac{2\sqrt{\pi t_D}}{C_{FD}\sqrt{A_{F1,\alpha}}}$	1/2
	$\alpha = 0,\ x_{eD} \neq 1$	$\lim_{\substack{t_D \to \infty \\ \alpha = 0}} p_{wD} = \frac{2\sqrt{\pi t_D}}{C_{FD}\sqrt{A_{F,0}}}$	
	$\alpha \neq 0.5,\ x_{eD} \neq 1$	$\lim_{\substack{t_D \to \infty \\ \alpha \neq 0.5}} p_{wD} = \frac{2\sqrt{\pi t_D}}{C_{FD} A_{F2,1,\alpha}}$	
	$\alpha = 0.5,\ x_{eD} \neq 1$	$\lim_{\substack{t_D \to \infty \\ \alpha = 0.5}} p_{wD} = \frac{2\sqrt{\pi t_D}}{C_{FD} A_{F2,0.5}}$	
	$\alpha \neq 0,\ x_{eD} = 1$	$\lim_{\substack{t_D \to \infty \\ \alpha \neq 0}} p_{wD} = \frac{2\sqrt{\pi t_D}}{C_{FD}\sqrt{A_{F3,1,\alpha}}}$	
	$\alpha = 0,\ x_{eD} = 1$	$\lim_{\substack{t_D \to \infty \\ \alpha = 0}} p_{wD} = \frac{2\sqrt{\pi t_D}}{C_{FD}\sqrt{A_{F3,0}}}$	

Early-Intermediate Time	$\alpha \neq 0.5,\ x_{eD} \neq 1$	$\lim_{\substack{t_D \to \infty \\ \alpha \neq 0.5}} p_{wD} = \frac{4\pi t_D^{1/4}}{\Gamma(1/4) C_{FD}\sqrt{A_{F2,1,\alpha}}}$	1/4
	$\alpha = 0.5,\ x_{eD} \neq 1$	$\lim_{\substack{t_D \to \infty \\ \alpha = 0.5}} p_{wD} = \frac{4\pi t_D^{1/4}}{\Gamma(1/4) C_{FD}\sqrt{A_{F2,0.5}}}$	
Late-Intermediate ($\alpha \to 1$) to Late ($\alpha \to 0$) Times	$\alpha \neq 0,\ x_{eD} \neq 1$	$\lim_{\substack{t_D \to \infty \\ \alpha \neq 0}} p_{wD} = \frac{2\pi t_D^{\frac{2-\alpha}{2}}}{(2-\alpha)\Gamma\left(\frac{2-\alpha}{2}\right) C_{FD} A_{F2,\alpha}}$	$\frac{2-\alpha}{2}$
	$\alpha \neq 0,\ x_{eD} = 1$	$\lim_{\substack{t_D \to \infty \\ \alpha \neq 0}} p_{wD} = \frac{2\pi t_D^{\left(\frac{2-\alpha}{2}\right)}}{(2-\alpha)\Gamma\left(\frac{2-\alpha}{2}\right) C_{FD} A_{F3,2,\alpha}}$	
Early-Intermediate ($\alpha \to 1$) to Late-Intermediate ($\alpha \to 0$) Times	$\alpha \neq 0,\ x_{eD} \neq 1$	$\lim_{\substack{t_D \to \infty \\ \alpha \neq 0}} p_{wD} = \frac{4\pi t_D^{\frac{2-\alpha}{4}}}{(2-\alpha)\Gamma\left(\frac{2-\alpha}{4}\right) C_{FD}\sqrt{A_{F2,\alpha}}}$	$\frac{2-\alpha}{4}$
	$\alpha \neq 0,\ x_{eD} = 1$	$\lim_{\substack{t_D \to \infty \\ \alpha \neq 0}} p_{wD} = \frac{4\pi t_D^{\left(\frac{2-\alpha}{4}\right)}}{(2-\alpha)\Gamma\left(\frac{2-\alpha}{4}\right) C_{FD}\sqrt{A_{F3,2,\alpha}}}$	
Intermediate to Late Times	$\alpha \neq 0.5,\ x_{eD} \neq 1$	$\lim_{\substack{t_D \to \infty \\ \alpha \neq 0.5}} p_{wD} = \frac{4\pi t_D^{(3-2\alpha)/4}}{(3-2\alpha)\Gamma\left(\frac{3-2\alpha}{4}\right) C_{FD} A_{F2,2,\alpha}}$	$\frac{3-2\alpha}{4}$
Intermediate to Late Times	$\alpha \neq 0.5,\ x_{eD} \neq 1$	$\lim_{\substack{t_D \to \infty \\ \alpha \neq 0.5}} p_{wD} = \frac{8\pi t_D^{(3-2\alpha)/8}}{(3-2\alpha)\Gamma\left(\frac{3-2\alpha}{8}\right) C_{FD}\sqrt{A_{F2,2,\alpha}}}$	$\frac{3-2\alpha}{8}$

Discussion of Results



Property	λ=20 ω=5.4E-4	λ=50 ω=2.2E-4	λ=250 ω=4.3E-5	λ=500 ω=2.2E-5
Matrix block dimension, h_m, ft	1.25	0.5	0.1	0.05
Natural fracture density, ρ_f, n_f/ft	0.8	2	10	20
Number of natural fractures, n_f	200	500	2500	5000

$$\omega = \frac{(\phi c_t)_m h_m}{(\phi c_t)_f h_f}$$

$$\lambda = 12\left(\frac{x_F^2}{h_m^2}\right)\left(\frac{k_m h_m}{k_f h_f}\right)$$