Modeling of Transient Flow of Diagnostic Fracture Injection Tests in Porous media Embedded with Discrete Fractures

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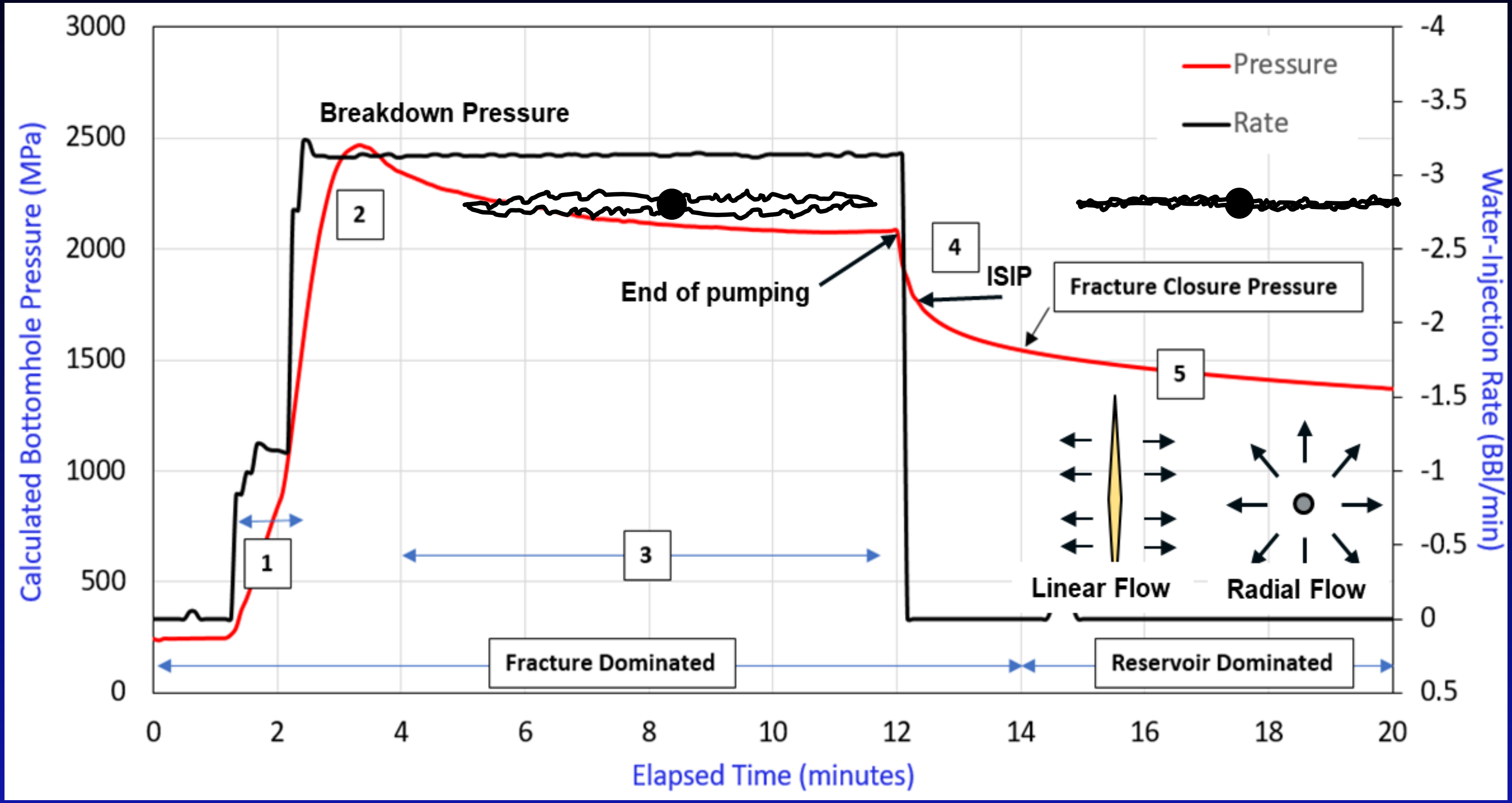
Outline

- Limitations of current methods
- Problem statement & motivation
- Geomechanics coupled reservoir flow simulation
- Semi-Analytical model development
- Semi-Analytical model development: Effect of NF
- Sensitivity analysis

Background



Pressure
Rate
Breakdown Pressure
End of pumping
ISIP
Fracture Closure Pressure
Linear Flow
Radial Flow
Fracture Dominated
Reservoir Dominated
1
2
3
4
5
3000
2500
2000
1500
1000
500
0
0
2
4
6
8
10
12
14
16
18
20
-4
-3.5
-3
-2.5
-2
-1.5
-1
-0.5
0
0.5
Calculated Bottomhole Pressure (MPa)
Water-Injection Rate (BBl/min)
Elapsed Time (minutes)

G-function Model

G-function developed by Nolte in 1979 based on caret's leakoff

$$v = \frac{C_L}{\sqrt{t}}$$

$\boldsymbol{v}$= Leak-off rate through unit area of fracture face
$\boldsymbol{C_L}$= leakoff coefficient controlled by filter cake
$\boldsymbol{t}$= length of time since the point of fracture has been exposed to the fracturing fluid.

Assumptions:

Negligible spurt loss

Power law fracture growth

Constant fracture surface area

Constant fracture compliance and

Carter's leak-off model (1D leak-off of fluid).

Mayerhofer Model

- In this model leakoff is decoupled into two major categories
  - Leakoff through fracture face (flow skin)
  - Linear flow of filtrate into formation

$$\Delta p(t_j) = \Delta p_r\left(t_j\right) + \Delta p_{face}\left(t_j\right)$$

- $\Delta p_r(t_j)$ pressure drop at any time in the reservoir
- $\Delta p_{face}\,(t_j)$ pressure drop at any time through filter cake

$$\Delta p_r = 141.2\frac{\mu q}{kh}\sqrt{0.000264\frac{\pi k t}{\mu \phi c_t x_f^2}}$$

$$P_D = \sqrt{\pi t_{Dx_f}} = \sqrt{\alpha_1 \frac{\pi k t}{\mu \phi c_t x_f^2}}$$

Mayerhofer Model

- The pressure change during pumping with varying fracture area and varying leakoff rate

$$\Delta p(t_m) = \left(\frac{2\times 141.2\ \mu_f \pi R_o}{A_{p,m}}\right) R_{D,m}\, q_{lm} + 4\times 141.2 \sqrt{0.000264 \frac{\pi\mu}{\phi k c_t}} \left[\sum_{j=1}^{m} \left(\frac{q_{lj}}{A_{p,j}} - \frac{q_{lj-1}}{A_{p,j-1}}\right) \sqrt{t_n - t_{j-1}}\right]$$

- Change of leakoff velocity during injection $\frac{q_{lj}}{A_{p,j}}$

Mayerhofer Model

- The pressure change during fracture closing is obtained by subtracting the pressure response during pumping from the total value during fracture closing

$$\Delta p(t_n) = [p_i - p(t_n)] - [p_i - p(t_m)]$$

$$\Delta p(t_n) = 4\times 141.2\sqrt{0.000264\frac{\pi\mu}{\phi k c_t}}\left[\sum_{j=1}^{m}\left(\frac{q_{lj}}{A_{p,j}} - \frac{q_{lj-1}}{A_{p,j-1}}\right)\sqrt{t_n - t_{j-1}}\right] + \left[\sum_{j=1}^{n}\left(\frac{q_{Fj}}{A_{p,j}} - \frac{q_{Fj-1}}{A_{p,j-1}}\right)\sqrt{t_n - t_{j-1}}\right] - \left[\sum_{j=1}^{m}\left(\frac{q_{lj}}{A_{p,j}} - \frac{q_{lj-1}}{A_{p,j-1}}\right)\sqrt{t_m - t_{j-1}}\right]$$

$$+\left(\frac{2\times 141.2\ \mu_f \pi R_o}{A_{p,m}}\right)(R_{D,n}q_{Fm} - R_{D,m}\, q_{lm})$$

- Change of leakoff velocity during Closing $\frac{q_{Fj}}{A_{p,j}}$
- Solved iteratively for $\Delta p(t_n)$ and t_n by use of $q_{Fj} = -c_f A_f [\frac{d\Delta p(t_j)}{d\Delta t_j}]$

Mayerhofer Model

- Satisfies the physics of filtration and linear elastic fracture mechanics while preserving the material balance.

- It is related to the physical properties R_o and k, but, unlike the G-function, it does not require knowledge of a constant leakoff coefficient at any time.

Mayerhofer Model

- Assumption of initial value of
  - *Fracture area* A_f
  - *Fracture face resistant* R_o
  - *K*
- Required for iterative computation

TABLE 1—OIL AND GAS RESERVOIR PRESSURE COMPONENTS

Input data	
Pumping time, minutes	20
Volume injected, bbl	400
PKN geometry	
x_f, ft	500
$h_f = h_p$, ft (at the end of pumping)	70
E', psi	4×10^6
β	0.75
Net shut-in pressure, psi	500
ϕ	0.1
μ_f, cp	0.5
c_t, psi^{-1}	
Oil reservoir	1×10^{-5}
Gas reservoir	2×10^{-4}
μ, cp	
Oil reservoir	1.0
Gas reservoir	0.022

Oil Reservoir

	R_0 (ft/md)	Δp_{face} (psi)	Δp_R (psi)	Δp (psi)
At k=0.1 md	0.3	284	8,766	9,070
	3	2,840	8,766	11,626
At k=1 md	0.3	284	2,778	3,062
	3	2,840	2,778	5,618
At k=10 md	0.3	284	878	1,162
	3	2,840	878	3,718

Motivation

- There are two folds for the problem proposed in this study
  - Approximate analytical approach to find a quick solution.
  - Claims of more accurate incorporating dynamic effect.

- Effect of randomly distributed natural fractures on the pressure transient during falloff ??

- It's crucial to understand the complete picture of the pressure transient responses in the natural fractures as well as the main fracture during the falloff.

Objectives of Study

- Develop a geomechanics coupled reservoir flow simulation for diagnostic fracture injection test.
- Allow the modeling of the pressure response of minifrac before fracture closure as well as the falloff period after closure of the fracture.
- The semi-analytical model simulates transient flow inside a homogenous porous media that contains finite conductivity, randomly connected and disconnected, natural fractures.
- Investigate effect of the main fracture as well as natural fractures properties, such a length, azimuth, conductivity and fracture distribution on the pressure and derivative response during falloff after injection.

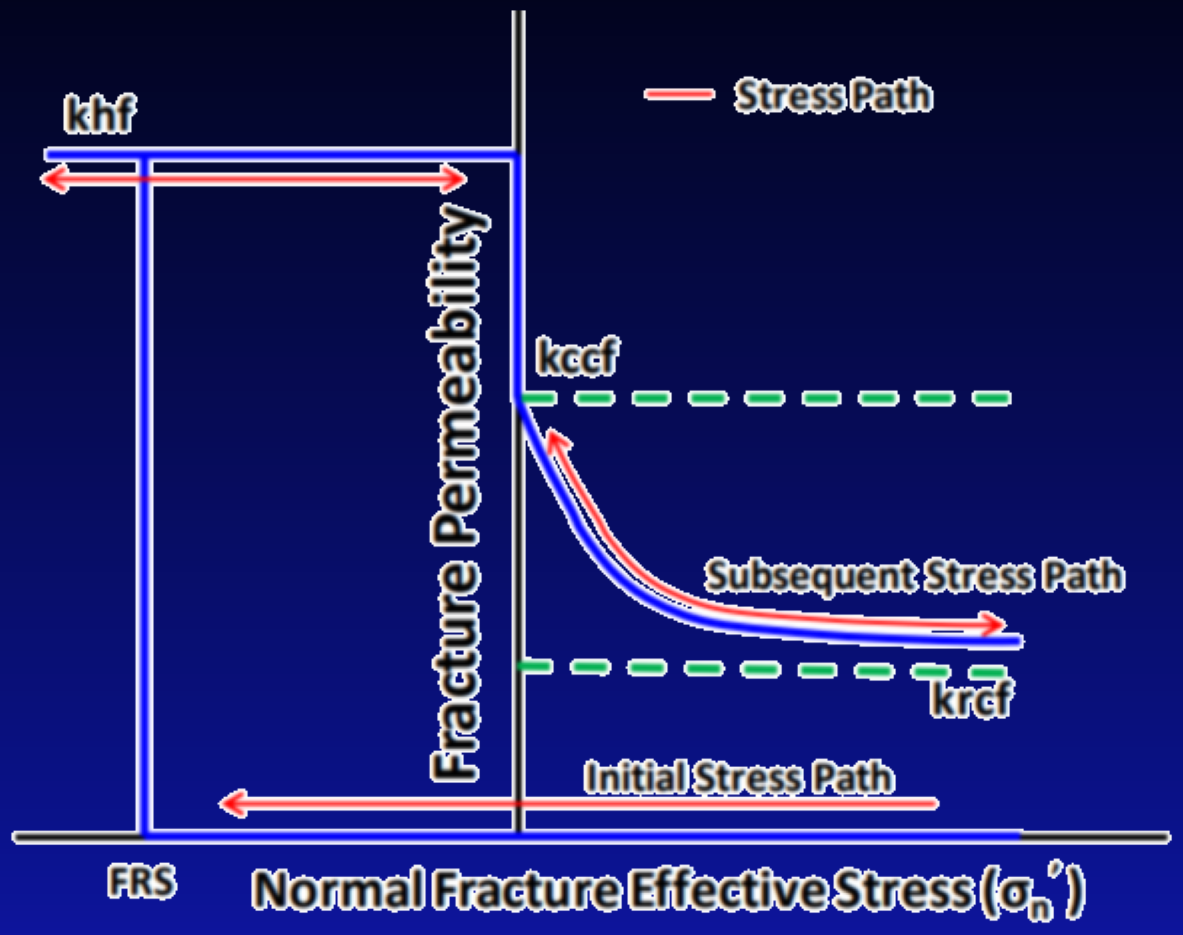
Geomechanics Coupled Reservoir Flow Simulation

Numerical Simulation

Geomechanics Coupled Reservoir Flow Simulation

- As pressure increases in the regular grid, the normal stresses on the fractures increase.

- Eventually the stress breaks past the failure envelope of the rock, causing a fracture to propagate and allow fluid to flow through the fracture system in addition to the underlying matrix system.



Barton-Bandis stress permeability relationship model

Geomechanics Coupled Reservoir Flow Simulation

- GEM is a numerical reservoir flow simulation tool with a coupled geomechanics feature that can model fracture initiation, propagation, closure, and falloff behavior of a typical minifrac.
- Barton-Bandis model is used to specify the relationship between the fracture opening and the permeability of the fracture system.
- In this model a secondary fracture system is defined in the grid via the standard dual permeability formulation.

Geomechanics Coupled Reservoir Flow Simulation

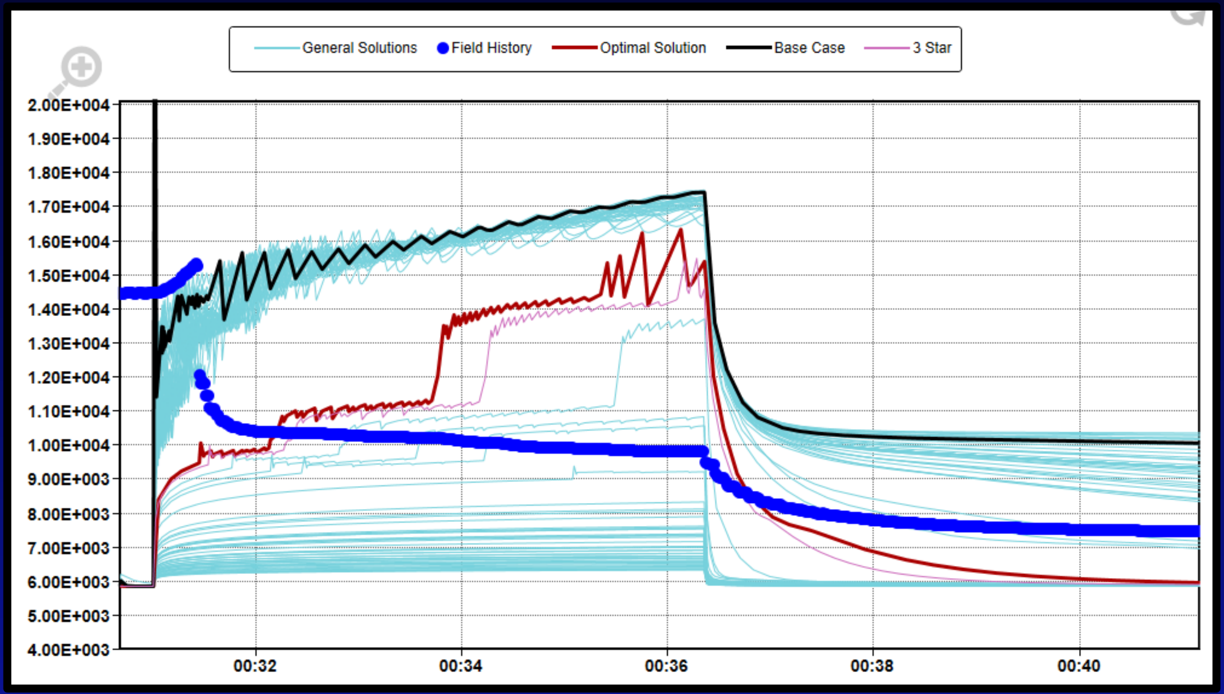
- Rock Mechanical Testing for geomechanical properties:
    - Mohr Failure Envelope (Failure strength)
    - Young’s Modulus
    - Poisson's ratio
    - Fracture opening stress
- Core analysis testing
    - Unconfined compressive strength UCS
    - Triaxial compressive strength under confined pressure

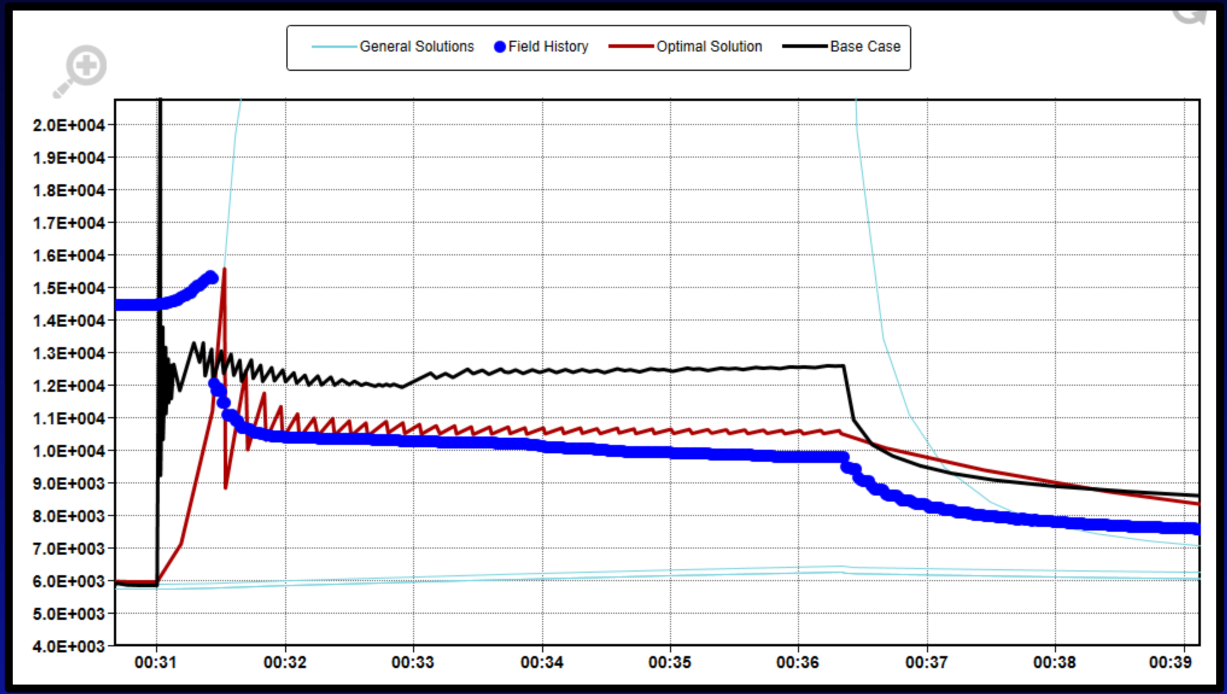
Geomechanics Coupled Reservoir Flow Simulation: Model Solutions

- Parameters and constraints
  - Fracture Closure Perm
  - Fracture Opening Stress
  - Residual Permeability
  - Natural Fracture Spacing
  - Permeability of Matrix
  - Permeability of Natural Fracture
  - Permeability of Hydraulic Fracture
  - Fracture Stiffness
- Objective functions to match
  - BHP

Geomechanics Coupled Reservoir Flow Simulation: Model Solutions

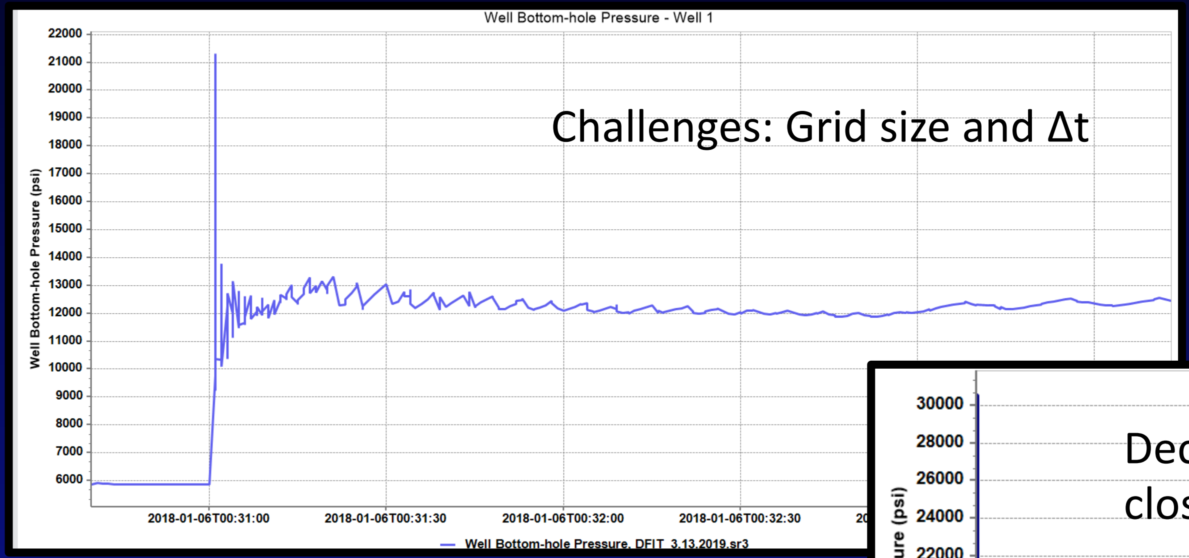
- Global error 8.84%

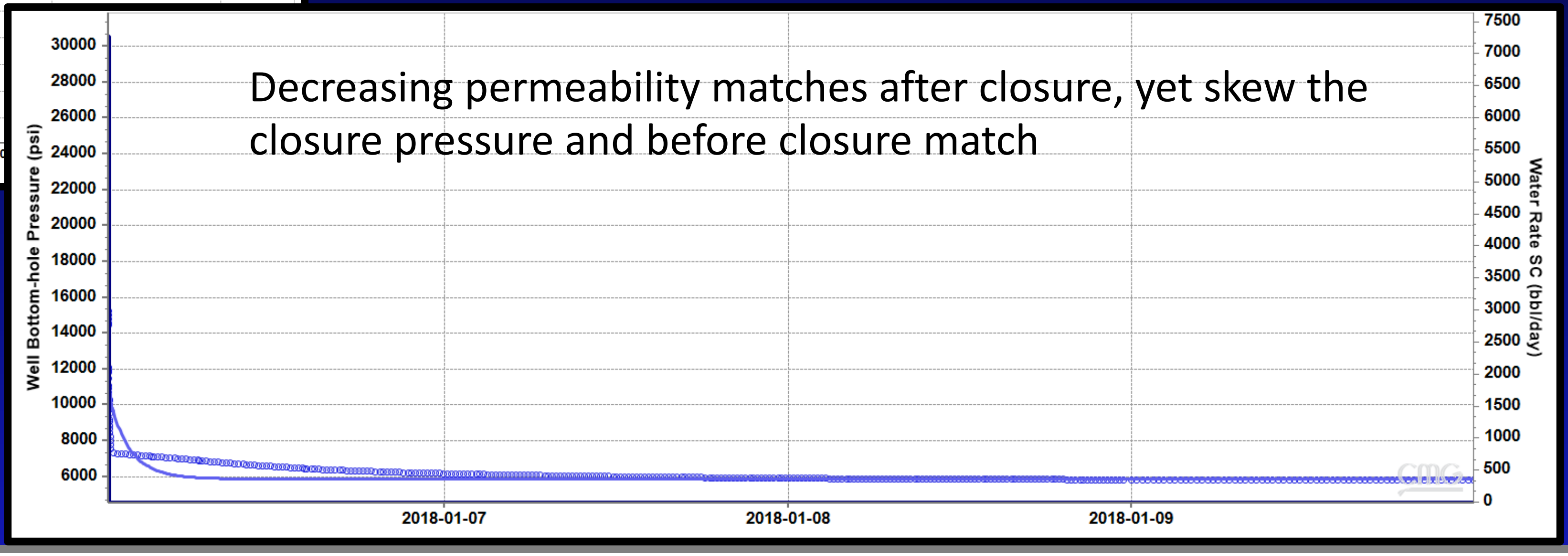




Geomechanics Coupled Reservoir Flow Simulation: Model Solutions

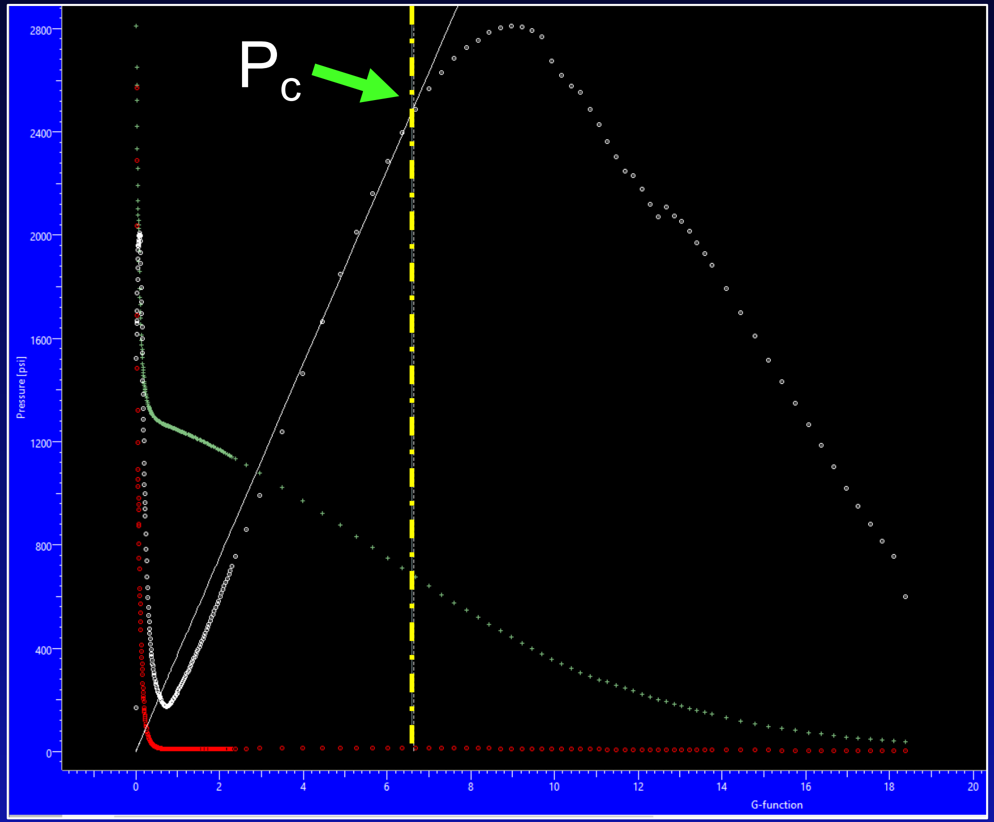
- Global error 8.84%



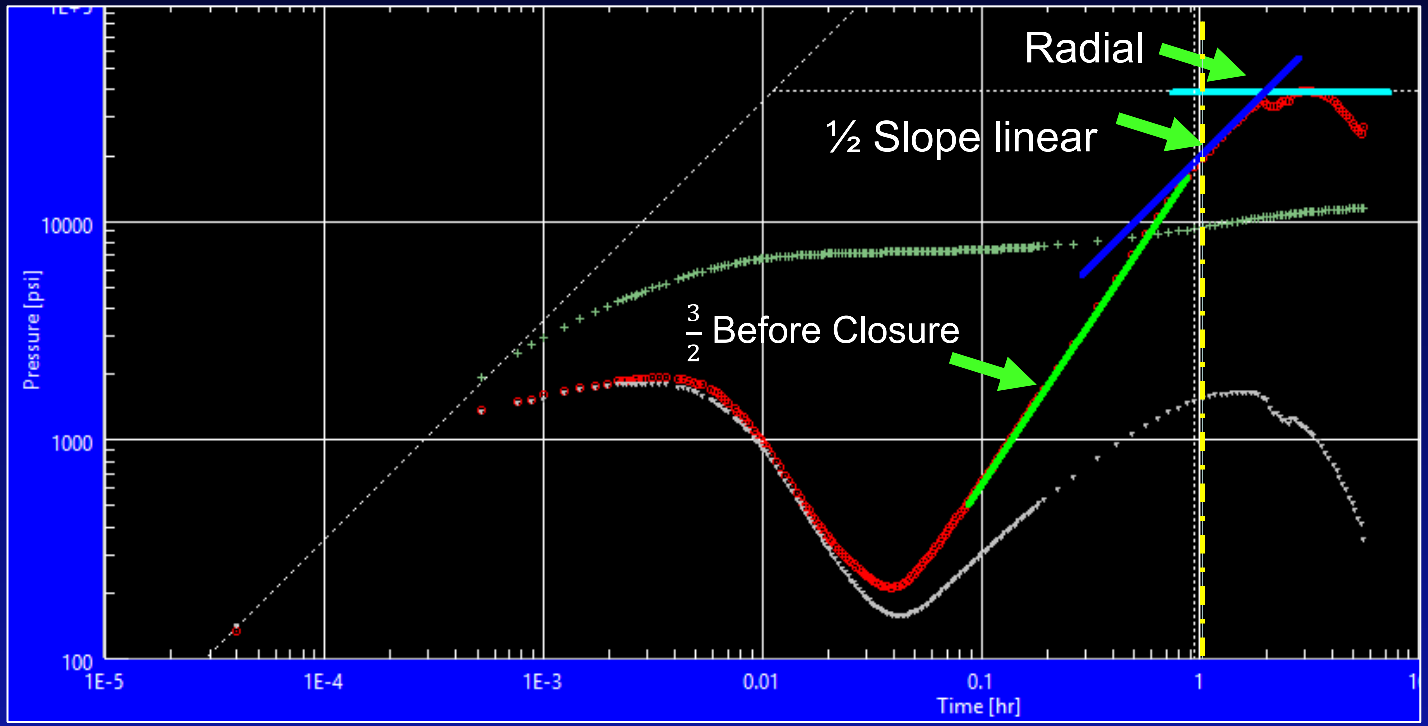


Geomechanics Coupled Reservoir Flow Simulation: Model Solutions

- DFIT results: CMG Modeling Compared to Field data



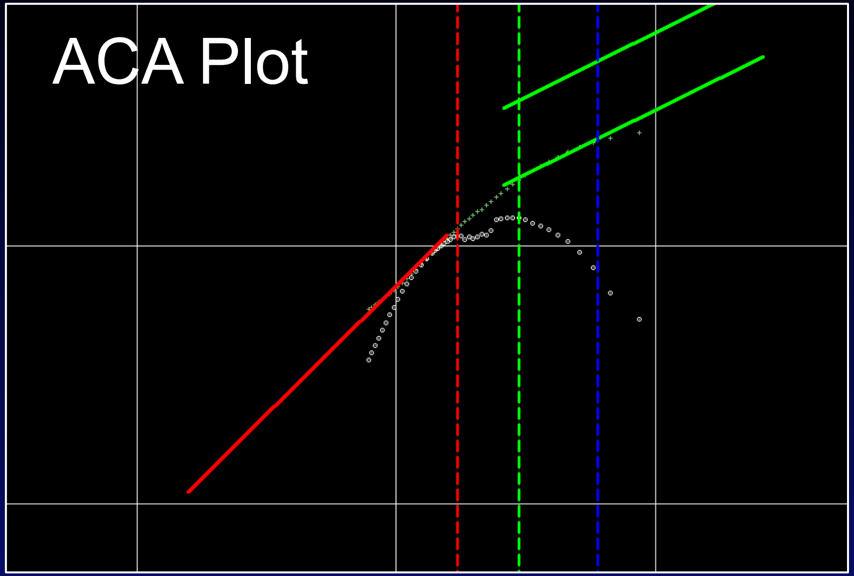
G-Function Plot



Log-Log Derivative Plot

Geomechanics Coupled Reservoir Flow Simulation: Model Solutions

- DFIT results: CMG Modeling Compared to Field data
  - Linear Flow was observed
  - Radial Flow was observed



	Geomechanics Coupled Reservoir Simulation	Field data
Initial Reservoir Pressure (Psi.)	5,407.0	5,644.6
Closure pressure (Psi.)	8,182.9	6,677.6
ISIP (psi.)	10,763.0	7,801.7
Kh, md.ft	4.14	1.65
Permeability, k (μd)	0.02	0.011

Pressure Transient in Homogeneous Porous Media

Semi-Analytical Modeling

Background on Previous Fluid Flow Models

Dual Porosity	Discrete-Fracture-Network Models
• May be extended to multiphase flow • Doesn't consider the complexity of fracture orientation • Assumes uniform distribution of the fractures through the reservoir	• Consider the complexity of fracture orientation • Ignore the flow from the matrix into isolated fractures

Semi-Analytical Model

Simulate flow inside a homogenous porous medium containing randomly distributed and unconnected fracture

Assumptions

- N_{nf} number of vertical NF (distributed arbitrarily and may intercept wellbore)
- An anisotropic and homogenous infinite slab reservoir
- Principle permeabilities of K_x and K_y coinciding with the cartesian coordinate system.
- Single phase flow under isothermal conditions
- Slightly compressible fluid of constant viscosity and compressibility
- Isolated and discrete NF
- 1D and incompressible fluid inside NF and HF
- Negligible gravitational force

Semi-Analytical Model

- Three decoupled models
  - Reservoir-flow model
  - Fracture-flow model (Izadi et al. 2007)
  - Fracture Propagation Geo-mechanical Model

- Coupled using continuity of mass and pressure at the fracture matrix interfaces.

Semi-Analytical Model: Reservoir-flow Model

$$k_x \frac{\partial^2 \Delta p}{\partial x^2} + k_y \frac{\partial^2 \Delta p}{\partial y^2} + S_w(t) + S_f(t) = \beta \frac{\partial \Delta p}{\partial t}$$

$$t = 0 \qquad \Delta p(0, x, y) = 0$$

1. $x \to \pm\infty$

$$\lim_{x \to \pm\infty} \Delta p(t, x, y) = 0$$

$$\lim_{x \to \pm\infty} \frac{\partial \Delta p}{\partial x} = 0$$

2. $y \to \pm\infty$

$$\lim_{y \to \pm\infty} \Delta p(t, x, y) = 0$$

$$\lim_{y \to \pm\infty} \frac{\partial \Delta p}{\partial y} = 0$$

S_w is the source function for vertical wells

$$S_w(t) = \gamma \sum_{m=1}^{M} q_{wm}(t)\delta(x - x_{wm})\delta(y - y_{wm})$$

S_f is the source function for both HF and NF

$$S_f(t) = \gamma \sum_{n=1}^{N_{tf}} \int_0^{L_{fn}} q_{fn}(u_n, t)\, \delta\left(x - x_{fn} - u_n \cos\theta_{fn}\right)\delta\left(y - y_{fn} - u_n \sin\theta_{fn}\right) du_n$$

Pressure drop at any point in the reservoir can be computed

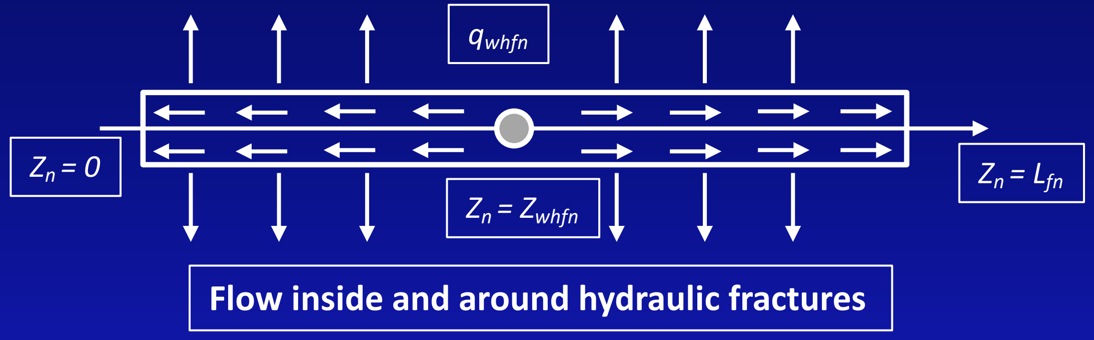
$$\Delta p(s, x, y) = C\left[\sum_{m=1}^{M} \tilde{q}_{wm}(s) K_0\left[a_{vom}\right] + \sum_{n=1}^{N_{tf}} \left\{ \int_0^{L_{fn}} \tilde{q}_{fn}(u_n, s) K_0\left[a_{tfn}(u_n)\right] du_n \right\}\right]$$

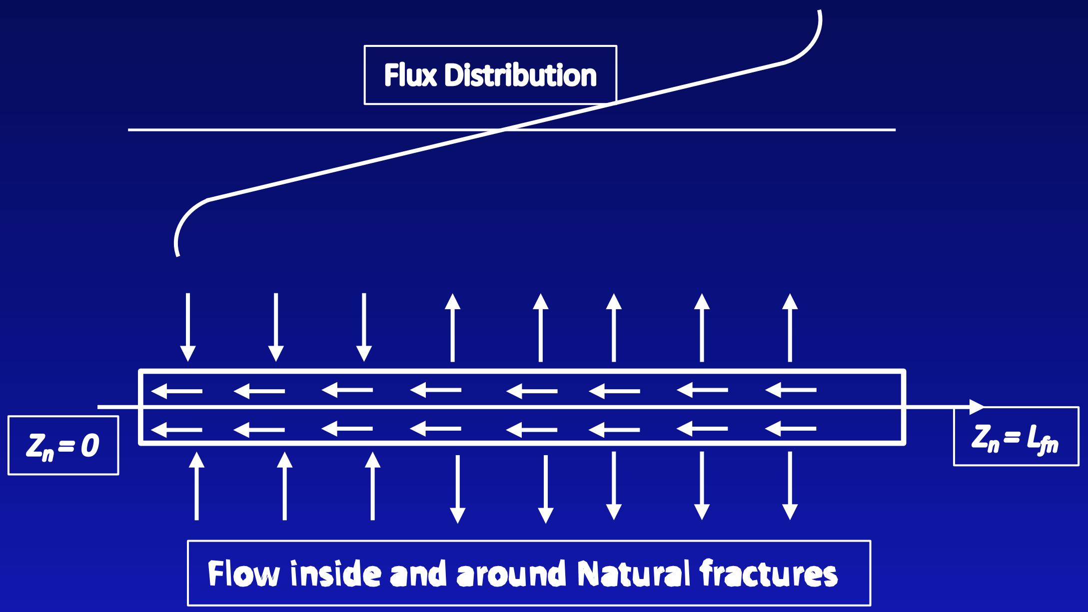
$$a_{vom} = \sqrt{\beta s}\sqrt{\frac{(x - x_{wm})^2}{k_x} + \frac{(y - y_{wm})^2}{k_y}}$$

$$a_{tfn}(u_n) = \sqrt{\beta s}\sqrt{\frac{\left(x - x_{fn} - u_n \cos\theta_{fn}\right)^2}{k_x} + \frac{\left(y - y_{fn} - u \sin\theta_{fn}\right)^2}{k_y}}$$

Semi-Analytical Model: Fracture-Flow Model

- All the fractures are assumed to be finite conductivity and incompressible
- 1D diffusivity equation with 2 source terms governs flow inside fractures
- Laplace transform in both time and space dimensions to develop fracture flow model.





Semi-Analytical Model: Fracture-Flow Model

- Fluid flow inside nth fracture in the system

$$\frac{\partial^2 \Delta p_{fn}}{\partial z_n^2} + \frac{q_{whfn}}{k_{fn}} \delta(z_n - z_{whfn}) - \frac{\mu B_o}{\alpha_1 k_{fn}} \int_0^{L_{fn}} \left\{ \frac{q_{sfn}(u_n, t)}{w_{hfn} h} \delta(z_n - u_n) \right\} du_n = 0$$

$$q_{sfn}(z_n, t) = \sum_{i=1}^{n} \frac{q_{sfi}(t)}{\Delta L_{fn}} \left\{ H\left[z_n - (i-1)\Delta L_{fn}\right] - H\left[z_n - i\Delta L_{fn}\right] \right\}$$

$$\rightarrow$$

$$\Delta \tilde{p}_{fn}(z_{skj}, s) = \Delta \tilde{p}_{fk}\Big|_{z_k=0} - \frac{1}{s}\frac{\gamma_{fk}}{k_{fk}} q_{whfk}(z_{skj} - z_{wfk}) + \frac{\gamma_{fk}}{k_{fk}} \left\{ \sum_{i=1}^{j-1} \tilde{q}_{fki}(s) \left(z_{skj} - z_{ski}\right) + \frac{\tilde{q}_{fkj}(s)}{\Delta L_{fk}} \frac{1}{2} \left(z_{skj} - (z_{skj} - \Delta L_{fk}/2\right)^2 \right\}$$

- Initial and boundary conditions

$$t = 0 \qquad \Delta p_f(z, 0) = 0$$

$$z_n = 0 \qquad \rightarrow \frac{\partial \Delta p_{fn}}{\partial z_n} = 0$$

$$z_n = L_{fn} \qquad \rightarrow \frac{\partial \Delta p_{fn}}{\partial z_n} = 0$$

- For Hydraulically fractured wells:

$$\sum_{i=1}^{N_{sn}} q_{sfi}(t) = q_{whfn}$$

- For Natural Fractures:

$$\sum_{i=1}^{N_{sn}} q_{sfi}(t) = 0$$

Semi-Analytical Model: Coupling Fracture- and Reservoir-Flow Models

- Using continuity of mass and pressure at the fracture matrix interfaces.
- The final solution for pressure at any point of the reservoir is in the form of a finite series containing fracture rates.
- Writing solution for all the fractures segments we end up with a matrix whose solution yields fracture pressure drop and the rate distribution along the fractures.

Semi-Analytical Model: Coupling Fracture- and Reservoir-Flow Models

$$\Delta\tilde{p}_{kj}(s)\big|_{Reservoir} = \Delta\tilde{p}_{fkj}(s)\big|_{Fracture}$$

$$\tilde{q}_{fkj}(s)\big|_{Reservoir} = \tilde{q}_{fkj}(s)\big|_{Fracture}$$

$\Delta\tilde{p}_{kj}(s)$ pressure drop at the fracture face on the reservoir side.

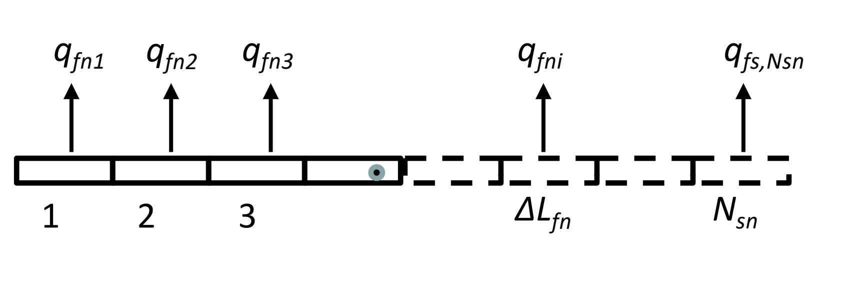
$\Delta\tilde{p}_{fkj}(s)$ pressure drop at the fracture face on the fracture side.

$\tilde{q}_{fkj}(s)\big|_{reservoir}$ Rate at which fluid leaves the reservoir at the face of the j^{th} segment on the k^{th} fracture.

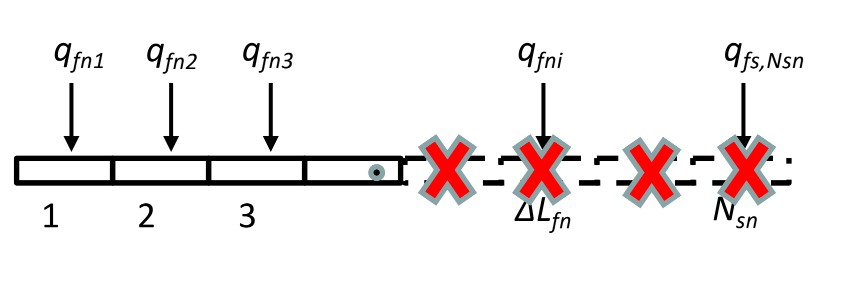
$\tilde{q}_{fkj}(s)\big|_{fracture}$ Rate at which fluid enters into the fracture at the face of the j^{th} segment on the k^{th} fracture.

Semi-Analytical Model: Geo-mechanical Model

- Fracture Propagation
  - Update fracture length and width at each time step
  - Use the new adjusted fracture length to calculate the pressure along the fracture.

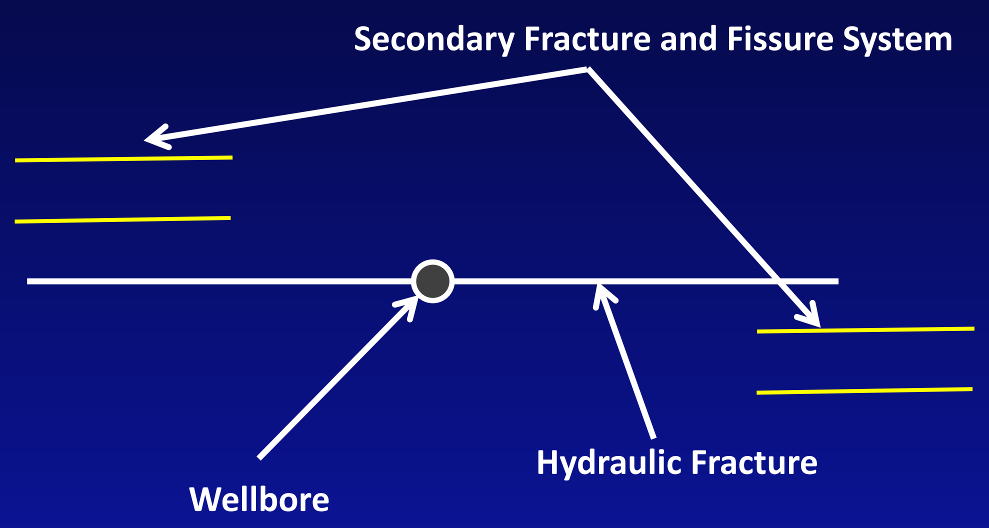


- Fracture Closure
  - Starting at end of injection, there is still flow rate inside the fracture.
  - Reservoir Pressure is higher than pressure inside fracture segments.
  - Segments with pressure less than the reservoir pressure, will be removed first.



Challenges with complex fractures and NF Distribution

- Effect of isolated fractures on the pressure transient behavior



- Randomly distributed but disjointed natural fractures from hydraulically fractured wells.