NUMERICAL SIMULATION OF POROELASTICITY AND MULTIPHASE FLOW IN NIOBRARA MATRIX-FRACTURE SYSTEM

by

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A thesis submitted to the Faculty and the Board of Trustees of the Colorado School of Mines in partial fulfillment of the requirements for the degree of Doctor of Philosophy (Petroleum Engineering).

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ABSTRACT

Production performance of unconventional reservoirs is affected by the coupled interaction between fluid depressurization and rock deformation. These reservoirs consist of a matrix, macro- and micro-fractures of which accurate representation and modeling of dynamic behavior of this multiple continua system are needed to model fluid flow. Transport equations for the fluid flow and rock deformations use two interacting environments consisting of a continuous fracture medium and a discontinuous rock matrix medium. A proper assignment of rock frame modulus, affected by the interconnected fractures, versus the rock matrix modulus is important to model long-term production performance of unconventional shale reservoirs.

This dissertation presents a new formulation for a numerical model that utilizes linear poroelastic theory and three-phase black oil approach in dual-porosity setting. Changes in pore pressure during production causes decrease in porosity, permeability, and reduction in the pore volume which, in turn, is reflected by an increase in pore compressibility. Most of the coupled geomechanics and flow simulation models do not account for such details in pore compressibility. It was observed that the energy provided by the reservoir compaction increases cumulative fluid production, and numerical models that have constant compressibility values in their transport equations underestimate the cumulative production.

Field data of Niobrara shale formation including 4D seismic survey, microseismic, core, well log, and production/pressure data provided by Reservoir Characterization Project (RCP) and Anadarko Petroleum Corporation is used to construct and validate simulation model. The theory behind the proposed model and its application to production data from hydraulically fractured horizontal wells in Niobrara shale formation will provide broader insights for the production-decline trend in unconventional reservoirs.
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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Fracture surface area, $L^2 \left(ft^2\right)$</td>
</tr>
<tr>
<td>$RB$</td>
<td>Reservoir barrel</td>
</tr>
<tr>
<td>$mD$</td>
<td>Millidarcy</td>
</tr>
<tr>
<td>$MSCF$</td>
<td>Thousand standard cubic feet</td>
</tr>
<tr>
<td>$B_g, B_o, B_w$</td>
<td>Formation volume factor for gas, oil, and water, $L^3/L^3 \left(RCF/SCF\right)$</td>
</tr>
<tr>
<td>$c_\phi$</td>
<td>Pore compressibility, $L^2/F^{-1} \left(1/psi\right)$</td>
</tr>
<tr>
<td>$\hat{c}_w, \hat{c}_g$</td>
<td>Compressibility of water and gas respectively, $L^2/F^{-1} \left(1/psi\right)$</td>
</tr>
<tr>
<td>$C_{loss}$</td>
<td>Fluid loss coefficient, $LT^{-1/2} \left(ft/\sqrt{min}\right)$</td>
</tr>
<tr>
<td>$C$</td>
<td>Fracture roughness measure and diagenetic factor (unitless)</td>
</tr>
<tr>
<td>$D$</td>
<td>Decline rate, $T^{-1} \left(day^{-1}\right)$</td>
</tr>
<tr>
<td>$D$</td>
<td>Depth, $L\left(ft\right)$</td>
</tr>
<tr>
<td>$E$</td>
<td>Young modulus, $FL^{-2} \left(GPa\right)$</td>
</tr>
<tr>
<td>$G$</td>
<td>Shear modulus, $FL^{-2} \left(GPa\right)$</td>
</tr>
<tr>
<td>$h$</td>
<td>Formation thickness, $L\left(ft\right)$</td>
</tr>
<tr>
<td>$h_g$</td>
<td>Height of the gas column, $L\left(ft\right)$</td>
</tr>
<tr>
<td>$h_w$</td>
<td>Height of the water column, $L\left(ft\right)$</td>
</tr>
<tr>
<td>$k$</td>
<td>Permeability, $L^2 \left(mD\right)$</td>
</tr>
<tr>
<td>$k_{f,eff}$</td>
<td>Effective permeability of the formation in the dual-porosity systems, $L^2 \left(mD\right)$</td>
</tr>
<tr>
<td>$k_f$</td>
<td>Micro/macro fracture permeability embedded in the matrix, $L^2 \left(mD\right)$</td>
</tr>
<tr>
<td>$k_m$</td>
<td>Matrix permeability, $L^2 \left(mD\right)$</td>
</tr>
<tr>
<td>$K_b$</td>
<td>Bulk or frame modulus, $FL^{-2} \left(psi\right)$</td>
</tr>
<tr>
<td>$K_{def}$</td>
<td>Bulk drained modulus of matrix blocks containing fractures, $FL^{-2} \left(psi\right)$</td>
</tr>
<tr>
<td>$K_{dm}$</td>
<td>Bulk drained modulus of matrix blocks without fractures, $FL^{-2} \left(psi\right)$</td>
</tr>
</tbody>
</table>
$K_{fl}$ Fluid modulus, $FL^2 (psi)$

$K_s$ Solid modulus, $FL^2 (psi)$

$K_{sm}$ Bulk modulus of solid minerals in the matrix, $FL^2 (psi)$

$L$ Hydraulic fracture length, $L (ft.)$

$m_l$ Special slope for linear flow regime, $1/2$

$m_{unit}$ Special slope for boundary-dominated flow regime, $1$

$M$ Biot modulus, $L^2/F^{-1} (1/psi)$

$M_f$ Biot modulus of the fracture medium, $L^2/F^{-1} (1/psi)$

$M_m$ Biot modulus of the matrix, $L^2/F^{-1} (1/psi)$

$n_{hf}$ Number of hydraulic fractures

$N_p(t)$ Cumulative oil production at time $t$, $L^3 (STB)$

$P$ Pressure, $FL^2 (psi)$

$p_f$ Pressure in the fracture, $FL^2 (psi)$

$p_m$ Pressure in the matrix, $FL^2 (psi)$

$p_{of}$ Pressure of the oil phase in the fracture, $FL^2 (psi)$

$p_{om}$ Pressure of the oil phase in the matrix, $FL^2 (psi)$

$p_{cwof}, p_{cogf}$ Water-oil capillary pressure and gas-oil capillary pressure in the fracture $FL^2 (psi)$

$q$ Rate, $L^3T^{-1} (BBL/D, MSCF/D)$

$\dot{q}$ Rate per unit rock volume, $L^3T^{-1} (BBL/D, MSCF/D)$

$R$ Actual fracture radius, $L (ft)$

$R_{so}$ Solution gas-oil ratio, $L^3/L^3 (SCF/BBL)$

$R_{sw}$ Solution gas-water ratio, $L^3/L^3 (SCF/BBL)$

$s_p$ Spurt loss, $L^3/L^2 (ft^3/ft^2)$

$SRV$ Stimulated reservoir volume, $L^3 (ft^3)$

$S_g$ Saturation of gas, (fraction)

$S_w$ Saturation of water, (fraction)

$t$ time, $T (day)$
$u_f$  Interstitial fluid phase velocity, $L/T$ ($ft/D$)

$u_s$  Interstitial solid phase velocity, $L/T$ ($ft/D$)

$v_f$  Darcy fluid velocity, $L/T$ ($ft/D$)

$V_p$  pore volume, $L^3$ ($ft^3$)

$V_b$  Bulk volume, $L^3$ ($ft^3$)

$w_f$  Fracture width, $L$ ($\mu m$)

$y_f$  Hydraulic fracture half- length, $L$ ($ft$)
LIST OF SYMBOLS-GREEK

\(\alpha\)  Biot coefficient
\(\alpha_f\)  Biot coefficient for the fracture
\(\alpha_m\)  Biot coefficient for the matrix
\(\varepsilon_v\)  Vertical strain
\(\phi\)  Porosity
\(\phi_f\)  Fracture porosity
\(\phi_m\)  Matrix porosity
\(\gamma_\beta\)  Specific gravity of the fluid
\(\lambda\)  Lame coefficient, \(FL^2\) (GPa)
\(\lambda_w, \lambda_o, \text{and} \lambda_g\)  Mobilities of water, oil, and gas, \(LT/m\) (1/cP)
\(\mu\)  Viscosity, \(m/LT\) (cP)
\(\nu\)  Poisson ratio
\(\rho_f\)  Fluid density, \(mL^{-3}\) (lb/cuft)
\(\rho_s\)  Solid density, \(mL^{-3}\) (lb/cuft)
\(\sigma\)  Shape factor, \(L^{-2}\) (ft^{-2})
\(\tau\)  Transfer function between matrix and fracture, \(T^{-1}\) (1/D)
\(\tau\)  Time, \(T\) (minute)
\(v\)  Velocity, \(LT^{-1}\) (ft/minute)
\(\zeta\)  Volumetric fluid content, \(L^3/L^3\) (cuft/cuft)
ACKNOWLEDGEMENTS

I want to express my sincere gratitude to my advisor Dr. Hossein Kazemi for his guidance and insight throughout the thesis study and during my studies in Colorado School of Mines. It has been a great privilege and pleasure to work with him, without his expert advice and support it is not possible to finish this work. I would also like to thank to my committee members, Dr. Ali Tura, Dr. Azra Tutuncu, Dr. Steve Sonnenberg, and Dr. Jennifer Miskimins.

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CHAPTER 1
INTRODUCTION

This dissertation presents reservoir characterization and production performance analysis of Niobrara shale reservoir using a newly formulated, fully coupled, geomechanical and fluid flow model. The research is sponsored by the Reservoir Characterization Project (RCP) in collaboration with Anadarko Petroleum Corporation (APC). The research integrates seismic, petrophysics, and production data in the model to determine the impact of rock deformation on reservoir performance in the Wishbone section. In what follows, I will present the details of the objectives and research contributions.

1.1 Motivation

Niobrara formation is one of the productive horizons in the Wattenberg Field, which is located in the Colorado portion of the Denver basin. The Wishbone section is the main study area of this work with hydraulically fractured horizontal wells, which is currently producing from Niobrara and Codell formations. The aim of current phase in this integrated multi-disciplinary project is to dynamically characterize the Niobrara shale reservoir by using time-lapse 9-component seismic survey, microseismic, core, well log, and production/pressure data.

Long-term production performance of unconventional reservoirs is affected by the interaction between fluid depressurization and rock deformation. The motivation of this dissertation is to accurately represent the complex reservoirs systems and assess the effect of rock elastic properties on well performance in shale reservoirs using a fully-coupled, dual-porosity, multi-phase simulation model.

1.2 Objectives

The objective of this thesis is to develop a reservoir model to address the dynamic behavior of the unconventional reservoirs during production. The unconventional shale reservoirs consist of matrix, macro/micro fractures, and hydraulic fractures. To account for the micro-fractures and the matrix media, a dual-porosity numerical model, which took into account the matrix-fracture deformation with time, was developed. The three-phase black oil numerical model utilized the
linear poroelastic theory (Biot, 1941). Results from fully-coupled geomechanics model could demonstrate the relation between elastic properties and stress-dependent rock parameters such as porosity, and permeability on reservoir performance. The results could also define the impact of factors to improve the accuracy of long-term production forecast. The theory behind the proposed model and its applications to production data from Niobrara wells provided broader insights for the production-decline trend in unconventional reservoirs.

1.3 Research Contributions

In this research, a new formulation to account for the rock deformation in a dual-porosity system, a fully-coupled three-phase numerical model was built. The model captured the reservoir heterogeneities from petrophysical and seismic measurements. Furthermore, the additional energy provided by the compaction drive using the ever-changing pore compressibilities for both fracture and matrix media was assessed. The model was built on production analysis using rate transient, decline curve, and an analytical hydraulic fracture propagation analysis for each well.
CHAPTER 2
RESERVOIR CHARACTERIZATION

Characterization of unconventional shale reservoirs is an integrated study where we need to acquire and combine data from different sources. These data include geological and petrophysical information from the study area, seismic and microseismic surveys supported by the PVT and daily production data from the horizontal wells.

This chapter begins by presenting the (1) geology of the Wattenberg Field, and stratigraphy of Niobrara and Codell formations (2) details of the study area, (3) and the list of available field data. The results of (4) core analysis and (5) fluid properties are also presented in this chapter.

2.1 Geology of Wattenberg Field

Denver basin is one of the largest basins in the Rock Mountain area; it comprises 70,000 square miles in eastern Colorado, southeastern Wyoming, and southwestern Nebraska (Figure 2-1). It is an asymmetric post-depositional structural basin with its axis paralleling and adjacent to the Front Range of Colorado and the Laramie Range of Wyoming. The deepest part of the basin is between Denver and Cheyenne with more than 13,000 feet of sedimentary sequence (Clayton and Swetland 1980).
Before Late Cretaceous, the Denver Basin was occupied by the Western Interior Seaway (WIS) which at maximum development extended from the Arctic Ocean to Gulf of Mexico (Figure 2-2), a distance of more than 3,000 mi and was over 1,000 mi from east to west (Kauffmann 1977). The Late Cretaceous to Early Eocene (65-40 Ma) Laramide Orogeny portioned the large Western Interior Basin into smaller basins including Denver, Piceance, and Powder River Basins (O’Neal 2015).
Cretaceous rocks of the Western Interior Basin (WIB) consist primarily of shale, siltstone, and sandstone. The regional stratigraphic cross-section in Figure 2-3 shows the facies relations across the central part of WIB (Molenaar and Rice 1988), where we can observe the hydrocarbon bearing zones investigated in this study; Niobrara and Carlile Formations (consists of Codell Sandstone).
Wattenberg field is located in the Denver Basin, Colorado between Denver and Greeley. It is one of the largest fields in the Rockies in terms of total proved reserves, areal extent and the number of wells drilled. Discovered at 1970, it is still one of the most active fields in the US. Developments in the horizontal well technology and multi-stage hydraulically fracturing techniques provides the continuous increase in the production of oil, gas and gas condensate. The study area is located approximately 35 miles northeast of Denver, CO (Figure 2-4) (RCP 2017).
Niobrara Formation and Codell Sandstone are one of nine horizons that are productive in
the field. Niobrara Formation consists of four chalk units with three intervening marl intervals.
The average thickness of the Niobrara formation is approximately 300 ft. The chalk units are
referred as the A, B, C, and Fort Hays chalks, where the B and C chalks are the main targets of
horizontal drilling by operators in the field (Sonnenberg 2015). The Codell Sandstone which is
in the upper section of the Carlile Formation is a gray, very-fine-grained bioturbated marine shelf
sandstone with a maximum thickness of 28 ft. (Weimer et al. 1986). Stratigraphic column of the
Denver Basin with details of the formations in this study illustrating the multiple pays is shown in
Figure 2-5 (RCP 2017).
Figure 2-5 Stratigraphic column for the Wattenberg area (Modified from Sonnenberg 2015)

Well log information is available from vertical wells within the Wattenberg area and horizontal wells in the Wishbone section. Figure 2-6 is an example type log from the Wattenberg area with gamma-ray, resistivity, compressional and shear sonic velocities. High resistivity intervals indicate B chalk, C Chalk, and Codell intervals (RCP 2017).
2.2 Study Area

Wishbone section is the main study area of this work with hydraulically fractured horizontal wells. This section, which is currently producing from Niobrara and Codell formations, is investigated by the Reservoir Characterization Project (RCP), in conjunction with Anadarko Petroleum Corporation. The area of interest is a fifty square mile area in the Wattenberg field, outlined by the regional seismic survey in Figure 2-7 (Brugioni 2017).
The Wishbone section contains 11 multistage hydraulically fracture horizontal wells, seven in the Niobrara Formation and four in the Codell Formation. Lateral spacing for the horizontal wells in the Niobrara section decreases as we move from East to West, and range from 600 to 1200 ft. Figure 2-8 shows the layout of the horizontal wells within the Wishbone section (RCP 2017).
2.3 Available Dataset

The field data available for this project is provided by Anadarko to Reservoir Characterization Project (RCP) for Phases XV and XVI. There are three seismic surveys which are outlined in Figure 2-7 (3D Regional Survey, 3D/3C Anatoli Survey, and Time lapse (4D)/9C Turkey Shoot). The time lapse survey area at Turkey Shoot also contains the Wishbone section where we have all daily production and pressure data from 11 horizontal wells. Additionally, we have tracer data, surface array microseismic, and core samples from nearby vertical wells, FMI image logs from horizontal wells in Wishbone section, and wireline logs that includes dipole sonic for the mechanical properties.

2.4 Core Data Analysis

To characterize the rock matrix parameters, several experiments such as unsteady state pressure decay permeameter, mercury injection capillary pressure (MICP), and GRI crushed core analysis were performed.

2.4.1 GRI Crushed Core Analysis

The samples were crushed to 20/35 mesh size to eliminate the effect of coring artifacts (such as microfractures) on matrix properties (Handwerger et al., 2011). All samples are cleaned by the Dean-Stark extraction. The crushed samples are vacuum dried until an equilibrium weight is achieved at 212 °F and then stored in a desiccator to mitigate capillary condensation. The crushed core analysis includes helium porosity, pressure decay permeability, and fluid saturations. This approach is convenient because of the use of fixed particle sizes for all samples and short measurement times.

The measured porosity values for each facies using GRI crushed core analysis was tabulated in Table 2-1.
Table 2-1 Porosity obtained from GRI crushed core analysis

<table>
<thead>
<tr>
<th>Facies</th>
<th>( \phi, % )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Marl</td>
<td>8.0</td>
</tr>
<tr>
<td>B Chalk</td>
<td>10.7</td>
</tr>
<tr>
<td>B Marl</td>
<td>7.5</td>
</tr>
<tr>
<td>C Chalk</td>
<td>9.5</td>
</tr>
<tr>
<td>C Marl</td>
<td>7.7</td>
</tr>
<tr>
<td>D Chalk</td>
<td>5.3</td>
</tr>
</tbody>
</table>

2.4.2 MICP Measurements

MICP measurements are used to obtain information on pore structure and pore throat size distribution. Mercury is injected at up to 60,000 psia in 118 incremental pressure steps. The contact angle and mercury surface tension used in the Washburn (1921) equation for pore-size calculations were 140 and 485 dynes/cm, respectively. MICP measurements were performed on dried (chip) samples by a commercial laboratory.

The first information obtained from MICP measurement can be obtained plotting the intrusion pressure versus the corresponding intrusion volumes on a semi-log plot. This plot provides the entry pressure as well as the intrusion trend. Twelve samples from Niobrara formation were used to conduct MICP measurements. For the twelve Niobrara samples, all the intrusion occurs at pressures above 10,000 psi except two samples (Nio3 and Nio4) (Figure 2-9) (Cho et al. 2016). These two samples (Nio3 and Nio4) from B Chalk have higher porosity and permeability than the other Niobrara samples since lower pressures were required to invade the pores implying that they had bigger pore throat sizes. This suggests that B Chalk of Niobrara is a better production zone. On the other hand, the rest of the ten samples with such a high entry pressure infers a lack of any connected porosity or that the connections were too small to allow mercury to enter since the mercury atom is approximately 0.3 nm in diameter, and the methane molecule is 0.4 nm in diameter.
The pore size distribution is obtained by calculating the changes in the slope of cumulative volume intrusion versus pore throat diameter. The shape of the incremental mercury saturation versus pore throat radius on a log-log plot provides the overview of porosity distribution. The average pore diameter is determined based on the highest peak from the plots of pore size distributions.

For Niobrara samples, two sets of pore throat radius were identified (Figure 2-10) (Cho et al. 2016). The radii vary significantly that B chalk (Nio3 and Nio4) is from 0.0419 μm to 0.0852 μm, and the other Niobrara samples are from 0.0046μm to 0.0119 μm. The average smaller pore throat radius was around 0.0035 μm, and that of the bigger pore throat radius was 0.04 μm.
The matrix permeability can be calculated using Kozeny-Carman equation (Equation 2-1) which shows the fundamental relation between porosity and permeability. This definition of permeability uses pore geometry, tortuosity, and porosity for engineering purposes:

\[ k = 10^3 \frac{r^2 \phi}{8 \tau} \quad (2-1) \]

Where \( k \) is permeability in \( \text{md} \), \( r \) represents a pore radius in \( \mu \text{m} \), and \( \tau \) tortuosity.

The geometric tortuosity is defined as the ratio of actual flow path length to the straight line of the porous medium (Equation 2-2). The relationship between porosity and tortuosity (Pirson, 1958) is:

\[ \tau = \phi^{1-c} \quad (2-2) \]

Where \( c \) is cementation factor

Cementation factor, which was first introduced by Archie (1942), is a function of shape and pore size distribution. For unconsolidated sand, the cementation factor is around 1.3 and for highly cemented rocks is around 2.2 (Poate et al., 2015).
The matrix permeability was calculated using Eq. 2-1 with input parameters from the MICP results for the two matrix pore throat sizes and assuming the cementation factor as 2.2. The matrix permeability for Niobrara samples was calculated as 1.26E-3 mD and 3.14E-6 mD using 10% porosity and 6% porosity (Kamruzzaman, 2015) for the bigger and smaller pore throats respectively.

2.4.3 Unsteady State Pressure Decay Permeameter

The CMS-300 apparatus, in the petroleum engineering department, is used to measure effective permeability values. The CMS-300 is an automated core measurement system that measures porosity and permeability of reservoir rock samples at each confining stress (Cho, 2012).

A Niobrara core sample from the B-Marl was used in the CMS-300 to measure matrix permeability of the core. The measured matrix permeability of the core, at net confining stress of 2,500 psia, was $10^{-1}$ to $10^{-2}$ mD shown in Figure 2-11.

![Figure 2-11 B Marl Matrix Permeability Hysteresis](image)

The matrix permeability measured from the pressure analysis of the Core Measuring System (CMS-300) is four orders of magnitude higher than the matrix permeability calculated from the MICP pore-size distribution. The difference between these two values is the presence of natural fractures (CT-Scan image, Fig. 2-12). The macro fractures embedded in the matrix increased the permeability measurement using CMS-300 whereas the MICP measurements were conducted using the crushed core samples, therefore, did not include these macro-fractures.
2.5 PVT Data Analysis

Oil and gas samples were collected from separator from the wells in Wishbone section to characterize the fluid properties. Then in the laboratories, the samples were recombined in a pressure-volume-temperature (PVT) cell using representative separator conditions (producing gas-liquid conditions of 1537 SCF/STB). Several PVT tests such as differential liberation, constant composition expansion, and saturates, aromatics, resins, and asphaltenes (SARA) were conducted.

The Niobrara and Codell formations are over pressured, the area has positive temperature anomaly, having temperature gradients up to 29 °F/1000 ft. in high GOR areas. Reservoir temperature is 240 °F and reservoir pressure is 4500 psia. The phase envelope of the fluid system with the constant liquid volume % at the reservoir condition is shown in Figure 2-13. The saturation pressure at reservoir temperature is 3729 psia.
Figure 2-13 Phase envelope for Wishbone section

The composition of the reservoir fluid (Table 2-2) constitutes mainly methane (CH\(_4\)), and ethane (C\(_2\)H\(_6\)) that is nearly 60\% of the molar composition is CH\(_4\) and C\(_2\)H\(_6\) which is the characteristic of oil production from shale formations and the reason for having low oil viscosity (Figure 2-15). There is no H\(_2\)S component and nearly 1.6 mole % of CO\(_2\).
Table 2-2 Reservoir fluid molar composition

<table>
<thead>
<tr>
<th>Component</th>
<th>Mole %</th>
<th>Component</th>
<th>Mole %</th>
</tr>
</thead>
<tbody>
<tr>
<td>N₂</td>
<td>0.249</td>
<td>C16</td>
<td>0.562</td>
</tr>
<tr>
<td>CO₂</td>
<td>1.571</td>
<td>C17</td>
<td>0.496</td>
</tr>
<tr>
<td>H₂S</td>
<td>0.000</td>
<td>C18</td>
<td>0.480</td>
</tr>
<tr>
<td>C₁</td>
<td>49.094</td>
<td>C19</td>
<td>0.424</td>
</tr>
<tr>
<td>C₂</td>
<td>11.566</td>
<td>C20</td>
<td>0.321</td>
</tr>
<tr>
<td>C₃</td>
<td>6.724</td>
<td>C21</td>
<td>0.281</td>
</tr>
<tr>
<td>iC₄</td>
<td>1.124</td>
<td>C22</td>
<td>0.254</td>
</tr>
<tr>
<td>nC₄</td>
<td>3.248</td>
<td>C23</td>
<td>0.222</td>
</tr>
<tr>
<td>iC₅</td>
<td>1.239</td>
<td>C24</td>
<td>0.196</td>
</tr>
<tr>
<td>nC₅</td>
<td>1.898</td>
<td>C25</td>
<td>0.172</td>
</tr>
<tr>
<td>C₆</td>
<td>2.333</td>
<td>C26</td>
<td>0.158</td>
</tr>
<tr>
<td>C₇</td>
<td>3.209</td>
<td>C27</td>
<td>0.139</td>
</tr>
<tr>
<td>C₈</td>
<td>3.506</td>
<td>C28</td>
<td>0.119</td>
</tr>
<tr>
<td>C₉</td>
<td>2.492</td>
<td>C29</td>
<td>0.109</td>
</tr>
<tr>
<td>C₁₀</td>
<td>2.002</td>
<td>C30+</td>
<td>0.805</td>
</tr>
<tr>
<td>C₁₁</td>
<td>1.361</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C₁₂</td>
<td>1.064</td>
<td>C30+ Mole Wt</td>
<td>509.84</td>
</tr>
<tr>
<td>C₁₃</td>
<td>1.004</td>
<td>C30+ Sp Gr</td>
<td>0.922</td>
</tr>
<tr>
<td>C₁₄</td>
<td>0.838</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C₁₅</td>
<td>0.738</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The high solution gas oil ratio (R<sub>so</sub>) above bubble point pressure obtained from Niobrara fluid samples are around 1882 SCF/STB (Figure 2-14) and it declines as gas comes out of solution below bubble point pressure (3729 psia).
The high oil compressibility (Figure 2-15) due to the high $R_{so}$, low fluid viscosities (Figure 2-16), and low fluid densities (Figure 2-18) assist the depletion drive production. The formation volume factor for both oil and gas are also shown in Figure 2-17.
Figure 2-16 Oil and gas viscosities

Figure 2-17 Formation volume factor for oil ($B_0$) and gas ($B_g$)
Figure 2-18 Fluid densities

The saturates, aromatic, resin, and asphaltenes (SARA) analysis was also performed to investigate the existence of asphaltenes in the crude oil. The SARA analysis also provides information about rough sorting of the crude oil constituents as well as the polarity of crude oil (Mullins 2007). There are two primary SARA screening criteria: colloidal instability (CII) and cross plot of saturates/aromatics versus asphaltene/resins. Asomaning and Watkinson (1998) defined the CII as the ratio of unfavorable fractions which are asphaltene and saturates to the favorable fractions which are resins and aromatics. If this ratio is greater than one, this implies the asphaltenes are likely to be unstable. Stankiewicz et al. 2002, proposed to cross plot the ratio of saturates/aromatics to the ratio of asphaltene/resins. Low ratio of asphaltene/resins is an indication of good colloidal stabilization.
The SARA fraction data is tabulated in Table 2-3. The SARA performed for Niobrara sample shows that there is no asphaltene precipitation.

Table 2-3 Wishbone SARA Analysis

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>weight %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saturates</td>
<td>79.83</td>
<td></td>
</tr>
<tr>
<td>Aromatics</td>
<td>15.34</td>
<td></td>
</tr>
<tr>
<td>Resins</td>
<td>4.83</td>
<td></td>
</tr>
<tr>
<td>Asphaltenes</td>
<td>0.00</td>
<td></td>
</tr>
</tbody>
</table>

The cross plot of the ratio of saturates/aromatics to the ratio of asphaltene/resins (Figure 2-19) shows that the Niobrara oil is stable.

The total acid number and water content, which are important for potential corrosion problems, are also analyzed (Table 2-4) and the results indicate that the risk of corrosion is very low.
Table 2-4 Summary of Liquid Properties in Pipeline

<table>
<thead>
<tr>
<th>Test</th>
<th>Method</th>
<th>Result</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Acid Number</td>
<td>ASTM D 664</td>
<td>&lt; 0.05</td>
<td>mg KOH/g</td>
</tr>
<tr>
<td>Water Content</td>
<td>ASTM D 4377</td>
<td>0.08</td>
<td>wt%</td>
</tr>
</tbody>
</table>
CHAPTER 3
PRODUCTION DATA ANALYSIS

The production behavior of wells from unconventional reservoirs provides insight to the transport mechanisms that affect well performance, reservoir characteristics and the factors affecting the production in such complex systems. Therefore, in the analysis of the production mechanism for Wishbone section the daily production data from all the wells in this area were investigated. The section consisted of eleven wells, of which seven were completed in the Niobrara formation, and four were completed in the Codell formation, and each well was stimulated with a large number of hydraulic fracture stages. This chapter presents an evaluation of production performance of individual wells and provides information about stimulation and completion data.

This chapter includes (1) daily production data analysis, (2) multi-phase rate transient analysis, and (3) decline curve analysis. The production data analysis will be integrated with the simulation results which will be shown in Chapter 6.

3.1 Production Profile Overview

There are eleven horizontal wells in the Wishbone section of Wattenberg field. After hydraulically stimulating the wells, all the wells put on production in September 2013. Of these eleven wells, four of them (Well 10C, Well 8C, Well 5C, and Well 3C) produce from Codell formation and seven of them (Well 1N, Well 2N, Well 4N, Well 6N, Well 7N, Well 9N, and Well 11N) produce from Niobrara formation (Figure 3-1).

Figure 3-1 Wishbone section horizontal wells (Modified from RCP 2017)
Multi-stage completions for all of the wells include sliding sleeve technique with at one well plug-and-perf technique was used to complete the well. In sliding sleeve technique, the sleeves are run as part of the casing string and may be cemented in place or externally isolated in the wellbore with hydraulic-activated or swellable open-hole casing packers (Soliman and Dusterhoft 2016). Plug and perf stimulation technique relies on the sequential pump-down conveyance and placement of plugs to temporarily isolate sections of the well to be hydraulically fractured (Aviles et al. 2015).

The commonly used technique to create more fractures with less fracturing spacing or multi-stage is zipper fracturing. Zipper frac technique enables fracturing of adjacent wells in sequence, i.e., any given pair of wells on the same pad is being fractured and hold the fracture pressure while the adjacent well is being fractured. While the second well is being stimulated, the net pressure created on the first well can divert the fracture direction and this, in turn, increases the stimulated reservoir volume (Belhadi et al., 2011). The three wells (Well 9N, Well 7N, and Well 8C) in Wishbone Section (Figure 3-1) were stimulated in pairs.

To evaluate the pressure and production decline, fluid (oil, gas, and water) rates and pressure profile were analyzed. It was observed that on average, pressure declined with a pressure drop of 1000 psi during the first six months and continued with near constant bottomhole conditions. The oil production was dropped to 80-100 BBL/D at the end of one-year production from initial production of maximum 400 STB/D after around a month of flowback period. The gas production started to increase to 1200 MSCF/D after two-and-half-months of production where the pressure dropped below the bubble-point pressure in most of the wells and then steadily declined to a value of 300 MSCF/D and continues as a plateau. The water production was high at initial production due to flowback period right after the stimulation and declined very rapidly to a value of 100 STB/D after one month of production, and the water-cut steadily dropped to a value around 5-10% after four years of production. For illustrative purposes, the trend of fluid rates and pressure decline for Well 1N is presented in Figure 3-2.
The cumulative oil and gas production over 1400 days varies across the Wishbone section. The cumulative oil and gas production for all wells in Wishbone section is shown in Figure 3-3a and Figure 3-3c, respectively. Most of the wells were completed with the same number of hydraulic fracture stages except two wells (Well 9N and Well 10C) which have fewer stages. Therefore, the cumulative oil and gas production were normalized by the stage number (Figure 3-3b and Figure 3-3d) to compare the production performance of individual wells.

Figure 3-2 Production and pressure history of Well-1N
The analysis of cumulative oil and gas production for all eleven wells in Wishbone section did not provide a distinctive result among the wells and formations. To decipher the production performance trends, gas-oil ratio (GOR) for all the wells were plotted. Figure 3-4 shows the producing GOR of all the Niobrara and Codell wells. The wells located in the west show higher GOR values which can be an indication of higher fracture intensity.
Figure 3-4 GOR behavior of eleven wells in Wishbone section

Microseismic data from Wishbone section (Figure 3-5) also indicates the increase in the number of microseismic events from east to west. This can explain the GOR increase from east to west as gas molecules are more mobile than oil molecules, the increased number of fractures in the western part helps gas molecules to move.

Figure 3-5 Microseismic events in Wishbone section (Grechishnikova 2017)
3.2 Decline Curve Analysis

The decline curve analysis has been used to predict the performance of oil and gas of reservoirs for both vertical and horizontal wells. Numerous empirical and analytical methods have been introduced for investigating the reservoir performances of unconventional reservoirs. All the methods aim to provide an equation for the flow rate decline during reservoir depletion while the operating conditions remain constant. This analysis will also be used for future production predictions of individual well performances.

3.2.1 Decline Curve Analysis Techniques

Decline curve analysis is defined as the empirical relation between flow rate and time to estimate the ultimate recovery for producing wells. Arps (1944) defined empirical mathematical relations for the exponential and hyperbolic decline.

The classical decline equation for the flow rate at any given time is:

\[ q(t) = q_i (1 + bD_i t)^{-1/b} \]  

(3-1)

Where, \( q(t) \) is the flow rate at any given time, \( q_i \) the initial flow rate, \( b \) the rate exponent, \( D_i \) the nominal decline rate at time zero, and \( t \) the time.

Assuming the existence of boundary-dominated flow regime, Arp provided the relation of hyperbolic decline for the rate exponent between 0 and 1, i.e. \( 0 < b < 1 \) in Equation 3-1.

For the exponential decline of flow rate, the mathematical description is shown in Equation 3-2 where it is assumed that rate exponent is zero, i.e. \( b = 0 \):

\[ q = q_i e^{-D_i t} \]  

(3-2)

In shale applications of decline curve analysis, it is observed that the rate exponent, \( b \), generally starts with a value of 4.0 for few days and declines gradually to a value of 2.0 for several months and approaches to a value of zero after around one year. These relations of rate exponent are very similar to flow rate analysis used in unconventional reservoirs. Thus, observing \( b = 4.0 \) is related to bilinear flow regime, \( b = 2.0 \) is related to linear flow regime, and \( b = 0 \) is related to boundary-dominated flow regime (Kazemi et al., 2015).
To forecast the future production, the decline curve analysis during boundary-dominated flow period is used as the basis for the rest for the future production. Finally, the estimated ultimate recovery (EUR) is defined as the sum of measured cumulative production and the estimated future production integrating Equation 3-2 from the onset of boundary-dominated flow regime to any future time:

$$N_p (t - t_0) = N_p (t_2 - t_0) + N_p (t - t_2)$$  \hspace{1cm} (3-3)

Where,

$N_p$ is the cumulative production, $t$ any future time, $t_0$ the initial time, $t_2$ the onset of boundary-dominated flow.

For black oil reservoirs, the SRV can be estimated using the cumulative hydrocarbon production within the SRV for multi-staged unconventional reservoirs during the entire life of the well (Kazemi et al., 2015):

$$\frac{N_p (t - t_0)}{n_{bf}} = SRV \left[ \left( \phi \frac{S_o}{B_o} \right)^0 - \left( \phi \frac{S_o}{B_o} \right)^n \right]$$  \hspace{1cm} (3-4)

Where,

$n_{bf}$ is the number of hydraulic fracture stage, $SRV$ the stimulated reservoir volume, $\phi$ porosity, $S_o$ oil saturation, $B_o$ oil formation volume factor, 0 denotes the initial time, $n$ denotes any time

It should also be noted that Equation 3-4 should be used as the hydrocarbon production, i.e. the cumulative of produced solution gas production if reservoir pressure drops below bubble point pressure and the cumulative of oil production.

### 3.2.2 Decline Curve Analysis in Wishbone

To analyze the well performances, decline curve analysis was performed for each horizontal well in the Wishbone section to investigate the success of hydraulic fracturing and the quantity of the stimulated rock volume being produced. By analyzing the production data of all the eleven wells, two different flow regimes were identified; transient flow and boundary-
dominated flow. After approximately 200-250 days, the horizontal wells reach the boundary, and the production of oil and gas was dominated by boundary. For illustrative purposes only Well 1N is shown here. Figure 3-6 is the flow rate for the total fluid production versus time in log-log plot for Well 1N. It was observed that around 250 days the fluid production was dominated by the boundary, i.e., SRV.

![Rate decline curve analysis for Well-1N](image)

To predict the well performance of each well, decline curve analysis using Arps empirical formulations was performed for the boundary-dominated flow region. Exponential decline analysis using Eq. 3-2 was used during this flow period.

The decline curve analysis for both oil and gas production for Niobrara formation (Well-1N) and Codell formation (Well-10C) were shown in Figure 3-7 a-d. For both wells, the choke sizes were gradually increased, and around 1000 days, the wells started to produce with fully open choke (it is denoted as 0/64” in this study). Therefore, the matching parameters for the early-time was different from that of each time when the well conditions were changed. The results for each time during boundary-dominated flow regime is tabulated in Table 3-1.
Oil production performance (Well-1N)

Gas production performance (Well-1N)

The Codell wells decline is sharper (Figure 3-6 c-d) than that of the Niobrara wells (Figure 3-6 a-b) for both oil and gas productions although similar operational changes were implemented that is the choke sizes were increased gradually to the same sizes at same times.
Figure 3-7 Decline curve analysis for oil and gas production during boundary dominated flow

The rate exponent, $b$, decreases as the flow reaches to the boundary dominated flow and eventually reaches to zero (Table 3-1).
Table 3-1 Decline Curve Analysis Results during boundary dominated flow

<table>
<thead>
<tr>
<th></th>
<th>Well-1N</th>
<th>WELL-10C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OIL</td>
<td>GAS</td>
</tr>
<tr>
<td></td>
<td>Boundary</td>
<td>Boundary</td>
</tr>
<tr>
<td></td>
<td>Early-time</td>
<td>Late-time</td>
</tr>
<tr>
<td>D (1/days)</td>
<td>0.0015</td>
<td>0.0012</td>
</tr>
<tr>
<td>b</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0.0027</td>
<td>0.0018</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The analysis of the decline curve indicated that the cumulative oil production during the transient flow were nearly as much as that of the early-part of the boundary-dominated flow for the two wells. The cumulative future oil production for 30 years was predicted using the last decline trend. It was observed that for both of the wells, half of the entire oil were produced during the transient flow and the rest of half were produced during the boundary-dominated flow period. The future oil production for the rest of the life of the well was comparably less than the previous oil production (Table 3-2).
### 3.3 Multi-phase Rate Transient Analysis

In unconventional reservoirs, well performances are generally analyzed using rate-normalized pressure drop due to the ever-changing flow rate and bottomhole pressure values during the well’s life depending on the operational conditions. The rate-normalized pressure drop analysis which is called Rate Transient Analysis (RTA) provides information about formation permeability and hydraulic fracture conductivity.

#### 3.3.1 RTA Techniques

The rate transient analysis for vertical wells in conventional wells was first introduced by Winestock and Colpitts (1965):

\[
\frac{\Delta p_{w_f}(t)}{q(t)} = 141.2 \frac{\mu}{kh} \left[ \frac{1}{2} \left( \ln t_D + 0.809 \right) + s \right]
\]  

(3-5)

Where, \( \Delta p_{w_f}(t) \) is the bottom-hole flowing pressure drop, \( q(t) \) flow rate, \( \mu \) viscosity, \( k \) formation permeability, \( h \) formation thickness, \( t_D \) dimensionless time, and \( s \) skin factor.

The RTA was studied by Wattenbarger et al. (1998) and Bello and Wattenbarger (2008), Nobakht and Clarkson (2012), and Tivayanonda (2012) for both unconventional oil and gas.
reservoirs. Uzun et al. (2016) presented both the single- and multi-phase flow for multi-stage hydraulically fractured, low-permeability, dual-porosity shale reservoirs. The rate-normalized pressure-drop equations for single- and multi-phase flow are presented in the following two equations, respectively.

\[
\frac{\Delta p_{nf}(t)}{q(t)B} = 4.064\frac{\mu}{\sqrt{k_{f,eff} n_{nf} h_{yf}} \left(\left(\frac{1}{(\phi_c)_{f,m}}\right)^{1/2}\right)} + 141.2 \frac{\mu}{k_{f,eff} h} s
\]

(3-6)

\[
\frac{\Delta p_{nf}(t)}{q_{total}(t)} = 4.064\sqrt{\frac{\sqrt{\pi}}{2}} \lambda_i^{-1}\left(\left(\frac{\lambda_i}{(\phi_c)_{f,m}}\right)\right)^{1/2} \sqrt{t} + \frac{141.2\lambda_i^{-1}}{k_{f,eff} h n_{hf}} s_{face}
\]

(3-7)

Where,

\[k_{f,eff} = k_f \phi_f + k_m\]

(3-8)

\[q_{total}(t) = q_o(t)B_o + q_g(t)B_g + q_w(t)B_w\]

(3-9)

\[\lambda_i = \lambda_o + \lambda_g + \lambda_w\]

(3-10)

\[\lambda_\varphi = \frac{q_\varphi(t)B_\varphi}{q_{total}(t)} \lambda_i ; \varphi = o, g, w\]

(3-11)

Here, \(\lambda_i\) is total mobility, \(q_{total}(t)\) total flow rate of the flowing phases, \(k_{f,eff}\) effective permeability of the formation in dual-porosity systems, \(k_f\) micro/macro fractures permeability imbedded in the matrix, \(\phi_f\) fracture porosity, \(k_m\) matrix permeability, \(q_o(t)\) oil flow rate, and \(B_o\) formation volume factor.

In this model, the flow hierarchy is assumed as follows: the matrix feeds the macro/micro fractures, these macro/micro fractures flow to the hydraulic fracture linearly, and eventually, the fluid flows to the horizontal well through the hydraulic fracture. In the RTA, the data is analyzed using the following procedure:
• Diagnostic plot (log-log) is used to identify the follow regimes
• Linear flow analysis is used to study linear flow regime and calculate formation permeability if linear flow regime is identified
• Bilinear flow analysis is used to study bilinear flow regime and calculate hydraulic fracture conductivity if bilinear flow regime is identified

3.3.2 RTA in Wishbone

The well performances of eleven wells in the Wishbone section were studied using multi-phase RTA technique. The multi-phase RTA for all the wells were consistent with the decline curve analysis explained in Chapter 3.2.2 such as two different flow regimes were identified; transient flow and boundary-dominated flow. For illustrative purposes, the multi-phase RTA results for Well-1N and Well-10C are shown here.

To identify the flow regimes, diagnostic plot, which is the rate-normalized pressure drop versus time in log-log scale, is used (Figure 3-8). The diagnostic plot shows two flow regimes for both wells: linear flow identified by ½ slope followed by the boundary-dominated flow identified by unit slope.

(a) Well-1N
Linear flow analysis (LFA) using multi-phase RTA was then implemented to calculate the effective formation permeability using Equation 3-7. The rate normalized pressure drop versus square root of time was plotted during the linear flow regime identified using diagnostic plot (Figure 3-9).

Figure 3-8 Diagnostic plot using multi-phase RTA
Figure 3-9 Linear Flow Analysis plot using multi-phase RTA

The slope of the linear flow analysis plot, the completion parameters (tabulated in Table 3-3), and the fluid and rock parameters (tabulated in Table 3-4) were used to calculate the product of effective formation permeability and the mobility.

Table 3-3 Input Parameters for LFA

<table>
<thead>
<tr>
<th></th>
<th>Well-1N</th>
<th>Well-10C</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_{1/2} ) (psi/STB/D/sqrt(day))</td>
<td>0.14</td>
<td>0.13</td>
</tr>
<tr>
<td>Stage count</td>
<td>32</td>
<td>20</td>
</tr>
<tr>
<td>( y_f ) (ft)</td>
<td>444</td>
<td>300</td>
</tr>
<tr>
<td>( k_{f,eff} \lambda_{tr} \frac{mD}{cP} )</td>
<td>0.0118</td>
<td>0.079</td>
</tr>
</tbody>
</table>
Table 3-4 Fluid and Rock Properties for the Wishbone Section

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_0$</td>
<td>2.168 RB/STB</td>
</tr>
<tr>
<td>$B_g$</td>
<td>1187 RB/MMSCF</td>
</tr>
<tr>
<td>$B_w$</td>
<td>1 RB/STB</td>
</tr>
<tr>
<td>$R_{so}$</td>
<td>1827 SCF/STB</td>
</tr>
<tr>
<td>vis oil</td>
<td>0.165 cp</td>
</tr>
<tr>
<td>vis gas</td>
<td>0.02 cp</td>
</tr>
<tr>
<td>vis water</td>
<td>0.5 cp</td>
</tr>
<tr>
<td>$h$</td>
<td>80 ft</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.1</td>
</tr>
<tr>
<td>$c_t$</td>
<td>0.00001 1/psi</td>
</tr>
</tbody>
</table>

The formation permeability results in Table 3-3 for both wells indicated that macro/micro fractures were successfully created when the effective formation permeability was compared with the measured matrix permeability in Chapter 2.4. Moreover, the y-intercept in Figure 3-9 was also the indication of successful stimulation.
CHAPTER 4
HYDRAULIC FRACTURING ANALYSIS

The well productivity is affected by reservoir heterogeneities, hydraulic fracture geometry, and the reservoir characteristics within the stimulated reservoir volume. To develop a better understanding of the connectivity between each horizontal well in Wishbone section, oil and water tracers was injected into one of the wells, and the fluid flow is tracked in the observation wells. This chapter presents tracer test analysis to shed light on the stimulation effectiveness and the inter-well connectivity of the stimulated reservoir volume. Because tracers test analysis alone does not enough to shed light on transport mechanisms, hydraulic fracture geometry, DFIT results should be used in addition to the tracer analysis.

Hydraulic fracture geometry is another factor that affects the well productivity in unconventional reservoirs. This study presents a practical approach to modeling hydraulic fracture geometry using well stimulation treatment data and rock physics data. The model is applied to the analysis of the eleven wells in Wishbone section to estimate the hydraulic fracture length and width for each well.

Diagnostic Fracture Injection Tests (DFIT) is a commonly used technique to evaluate reservoir properties, fracturing parameters and obtain in-situ stresses. It corresponds to a single cycle composed of one injection and one decline. The DFIT is performed after the well has been cemented and before the main hydraulic fracture stimulation to obtain the in-situ primary parameters that control the volume and geometry of the fracture which is important to design the hydraulic fracture stimulation.

This chapter includes (1) tracer data analysis, (2) fracture propagation, and (3) DFIT analysis. The fracture propagation data analysis will be integrated with the simulation model which will be shown in Chapter 6.

4.1 Chemical Tracer Data Analysis

One of the most commonly used technique to obtain information about the reservoir heterogeneity and inter-well connections is tracer test. A tracer is a chemical or another substance
which is injected into a well and monitored the reaction while collecting the samples from the producer wells. In unconventional reservoirs, tracers are used to provide information about the contribution from each stage over time (Catlett et al., 2013), flowback efficiency, and geological interpretations through the static models.

### 4.1.1 Tracer Types

In unconventional reservoirs, the tracers are monitored both in the injector and neighboring producer wells. During flowback period, the injector wells are produced, and the injected tracers are monitored from the same well, which gives information about the flowback efficiency. Also, these tracers are monitored from the nearby producer wells which provide information regarding inter-well communications. There are three types of tracers: radioactive, chemical, and dye tracers.

Radioactive isotope tracers are injected. Most commonly used radioactive tracers are Scandium ($^{46}$Sc), Antimony ($^{124}$Sb), Iridium ($^{192}$Ir), Bromine ($^{82}$Br), and Iodine ($^{131}$I and $^{125}$I) according to Nuclear Regulatory Commission (NRC). The movement of the mixture including the radioactive tracer is traced by gamma ray detectors in the spectral gamma-ray log (SLB Oilfield Glossary). One of the main advantages of using radioactive tracers is that they are detectable even at very low concentrations. Thus minimal quantities can be injected. Other main advantages are that they are not adsorbed on the rock surface, and their reaction are not affected by reservoir conditions such as rock formation, pressure, and temperature (Zecheru and Goran, 2013). On the other hand, the gamma-ray logging tool can detect only two ft. in the formation (SLB Wireline Services Catalog). Therefore, the measurement scale is low compared to the other two tracer types.

Chemical tracers are divided into two types: oil-based and water-based chemical tracers. In unconventional reservoir application, a specific injection protocol is followed. The water-soluble fracturing fluid tracers are injected as a mixture of fracturing fluid. The water-dispersible oil tracers are injected just before injecting proppant, and both tracers propagate into the reservoir. When the injected fluid contacts to the reservoir oil, the oil tracers are partitioned into the contacted oil and produced together with the oil particle.

The dye tracers, such as fluorescein and the B rhodamine, are detected using spectrofluorimeter. The dye tracers are generally used for the injector-producer communication analysis in highly-fractured reservoirs. The detection of very low concentration is the main
advantage of dye tracers. However, it should be noted that they are adsorbed on the rock (Zecharu and Goran, 2013).

4.1.2 Tracer Application in Wishbone

To identify the reservoir heterogeneity in Wishbone section, both oil-based and water-based chemical frac tracers (CFT’s) were injected (Figure 4-1). Specifically, the communication between Codell and Niobrara formations and the contribution from the stages for each well were studied.

![Figure 4-1 Wishbone section horizontal wells tracer types](image)

Water-based CFTs are injected into the all eleven wells using fourteen different compositions (Figure 4-2), and oil-based CFTs are injected in two wells: Well-2N (three different compositions) and Well-6N (four different compositions) (Figure 4-3).
<table>
<thead>
<tr>
<th>TOE</th>
<th>1N</th>
<th>1C</th>
<th>2N</th>
<th>3N</th>
<th>4N</th>
<th>5N</th>
<th>6N</th>
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</thead>
<tbody>
<tr>
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<td>1100</td>
<td>1000</td>
<td>1000</td>
<td>1100</td>
<td>1100</td>
<td>1100</td>
<td>1100</td>
<td>1100</td>
<td>1100</td>
<td>1100</td>
</tr>
</tbody>
</table>

Figure 4-2 Water-based tracers (Dang 2016)

<table>
<thead>
<tr>
<th>TOE</th>
<th>1N</th>
<th>1C</th>
<th>2N</th>
<th>3N</th>
<th>4N</th>
<th>5N</th>
<th>6N</th>
<th>7N</th>
<th>8N</th>
<th>9N</th>
<th>10N</th>
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</thead>
<tbody>
<tr>
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<td>5330</td>
<td>5630</td>
<td>5100</td>
<td>5200</td>
<td>5000</td>
<td>5500</td>
<td>1300</td>
<td>2700</td>
<td>2400</td>
<td>1500</td>
</tr>
</tbody>
</table>

Figure 4-3 Oil-based tracers (Dang 2016)
The concentration of each tracer in all eleven wells was identified and measured by collecting the fluid during flowback, and the recovery degree on the returning fluid is calculated (Figure 4-4). It is observed that in all eleven wells around ten percent of the injected fluid is recovered during flowback period of 180 days.

Figure 4-4 Tracer return versus elapsed flowback time for all eleven wells

The same type of water-based CFTs is injected in different wells (Figure 4-2) which led the interpretation of water-based CFTs difficult. Studying all eleven wells, it is observed that water-based CFTs return from the toe is higher than the other areas because of the lack of micro/macro fractures. The opposite is observed in the heel section (Figure 4-5 and 4-6).

The oil-based tracer return is higher in the Graben area than that of the water-based tracer returns. This could imply that there are more open fractures in the Graben area (Figure 4-5 and 4-6).
(a) Water Based Tracers

(b) Oil Based Tracers

Figure 4-5 Fracture intensity distribution from tracer analysis
The increase in the fracture intensity around the graben can be observed in the study performed by Grechishnikova 2016 in Figure 4-6a and Figure 4-6b in which the fracture intensities for each horizontal well was determined by the image log analysis.

(a) Total fracture intensity and lithofacies model along 2N well (Grechishnikova 2017)

(b) Total fracture intensity and lithofacies model along 6N well (Grechishnikova 2017)

Figure 4-6 Fracture intensity distribution from image logs
The analysis of oil-based CFTs is used to identify the inter-well communication as well as inter-layer communication where oil-based CFTs are injected from one well (Well-2N and Well-6N) and the samples from the neighboring wells were monitored.

There were three different compositions of oil-based tracers were injected from the toe, center, and the heel in Well-2N. The monitoring wells are Well-4N, Well-1N, and Well-3C (Figure 4-7).

![Figure 4-7 Oil tracer injection in well 2N](image)

It is observed that all three different compositions of oil-based tracers were collected from Well-3C which indicates the communication between Codell and Niobrara formations. The ranking by tracer recovery in the offset wells is Well-3C, Well-4N, and Well-1N. This implies that the strongest communication is between inter-layers (Well-2N and Well-3C) than the inter-wells.

Similar observations can be made by analyzing the water-based CFTs injected from the offset wells (Well-4N, Well-3C, Well-1N, Well-5C, and Well-6N) and monitored the recovery in Well-2N (Figure 4-8). The largest communication is observed between Well-2N and Well-3C which confirms the vertical layer communication.
Figure 4-8 Water tracer results for well 2N

For Well-6N, four different compositions oil-based tracers were injected from the toe, Center 1, Center 2, and the heel (Figure 4-9). The four offset wells (Well-7N, Well-8C, Well-5C, and Well-4N) are monitored for the returns to identify the communications between the wells (Figure 4-10).

Figure 4-9 Oil tracer injection in well 6N

All four compositions of oil-based tracers were collected in Well-5C and Well-8C which confirms the inter-layer connection between Codell and Niobrara formations. The ranking of the wells by tracer recovery in the offset wells is: Well-5C, Well-7N, Well-4N, and Well-8C.

The analysis of water-based CFTs which were injected from the offset wells and collected from Well-6N yields another possible ranking for the strongest to weakest communication as Well-
7N, Well-5C, Well-8C, Well-3C, and Well-4N. This also indicates that water-based CFTs and oil-based CFTs do not necessarily follow the same path which neither reduces the uncertainty in the flow path nor explains the transport mechanisms in the reservoir.

Figure 4-10 Water tracer results for well 6N

4.2 Fracture Propagation

The geometry of the hydraulic fractures affects well productivity in unconventional shale reservoirs. Modeling hydraulic fractures require simplifying assumptions because of the complexity of the rock deformation and the quality of rock physics data available. Nonetheless, modeling hydraulic fractures provides insight into the well productivity of unconventional reservoirs. In this section, I will present a practical approach to modeling and estimating hydraulic fracture length and width using well stimulation treatment data of the horizontal wells in the Wishbone.

4.2.1 Hydraulic Fracture Modeling for Well Performance Analysis

In hydraulic fracture modeling, we generally assume the fracture geometry based on laboratory observations and invoke force balance on the conceptual physical model to arrive at governing equations. The model presented in this study relies on fracture propagation theory of Perkins, Kern, and Nordgren (PKN) and utilizes acoustic log rock mechanical properties and fracture treatment data for each stage.
4.2.1.1 Literature Review

Petroleum industry has used hydraulic fracturing since the early fifties to stimulate production from low permeability reservoirs (Fjær et al., 2008). Measuring hydraulic fracture dimensions is difficult; therefore, a theory that can accurately model fracture propagation is essential. It is also essential to know the relationship between injection rate, proppant concentration, injected volume, and fracture length and width. The mathematical theory of crack-like defects goes back to Griffith (1921). Later Sneddon (1946) showed that fracture created by a constant internal pressure \( p \) has an elliptical shape, and provided an equation (Equation 4-1) to calculate the maximum fracture opening \( w_f \).

\[
\frac{w_f}{L} = \frac{4(1-\nu^2)pL}{E}
\]  
(4-1)

Where, \( \nu \) is Poisson ratio, \( E \) Young modulus, and \( L \) fracture length.

Mathematical formulations by Perkins and Kern (PK) (1961) and Geertsma-de Klerk (GdK) (1969) are two well-known fracture propagation models (Figure 4-11). Both models simulate fracture as a 2D, penny-shaped vertical fracture of constant height. PK calculates fracture width \( w_f \) in feet via Equation 4-2:

\[
w_f = 0.0272 \left[ \frac{(1-\nu)q\mu L}{G} \right]^{1/4}
\]  
(4-2)

Specifically, in Equation 4-2, \( w_f \) is fracture width in feet, \( q \) total injection rate in BBL/min, \( \mu \) fluid viscosity in cP, and \( G \) shear modulus in psi. Equation 4-3 relates shear modulus to Young modulus \( E \) and Poisson ratio \( \nu \):

\[
G = \frac{E}{2(1+\nu)}
\]  
(4-3)

Nordgren (1972) extended PK model by taking into account the fluid leak-off into the formation; hereafter, we will designate the Nordgren model as PKN model. In incorporating fluid leak-off during fracture propagation, Nordgren used the material balance formulation by Carter...

Figure 4-11 Perkins-Kern-Nordgren (PKN) model (a), Geertsma-de Klerk (GdK) model (b)

4.2.1.2 Mathematical Formulation

The PKN model assumes vertical plane strain \((\epsilon_x \neq 0, \epsilon_y \neq 0, \epsilon_z = 0)\). This leads to longer and narrower fractures. However, GdK assumes horizontal plane strain \((\epsilon_x \neq 0, \epsilon_y \neq 0, \epsilon_z = 0)\), resulting in wider fractures and shorter fracture lengths. Therefore, GdK model is more appropriate for situations where fracture height is larger than fracture length as in gravel packing remediation. In thin unconventional reservoirs, hydraulic fractures probably propagate to a longer length compared to the bed thickness (thus, the fracture height). Therefore, for our area of investigation we used the PKN model to calculate the length and the width of the hydraulic fracture.

Carter (1957) used the material balance between the injected fracturing fluid flow rate versus the rate of fluid loss to the formation and the fluid rate that created the fracture (Equation
Where, $q_{inj}$ is the injected fluid rate, $q_l$ fluid-loss rate, and $q_f$ the fracture expansion rate. The latter two flow rates are related to fracture volume expansion as shown below:

$$q_l(t) = 2 \int_0^{A(t)} v(t - \tau) dA$$  \hspace{1cm} (4-5)

$$q_f(t) = (w_f + 2s_p) \frac{dA(t)}{dt}$$  \hspace{1cm} (4-6)

Where, $v$ is the velocity of leaking filtrate at time $t$, $A(t)$ the fracture surface area for one wing of the fracture at time $t$ and $s_p$ the spurt loss. Furthermore, $t - \tau$ is the incremental exposure time of the fluid to leakoff, $t$ total time, and $\tau$ exposure time of the fracturing fluid to the fracture tip.

For a two-wing fracture, the material balance is:

$$q_{inj} = 2 \int_0^{A(t)} v(t - \tau) dA + (w_f + 2s_p) \frac{dA(t)}{dt}$$  \hspace{1cm} (4-7)

Eq. 4-8 expresses the leakoff velocity $v$ into the formation:

$$v(t) = \frac{C_{loss}}{\sqrt{t}}$$  \hspace{1cm} (4-8)

To solve for the fracture surface area in Equation 4-7, one can use Laplace transform:

$$\mathcal{L} \{ q_{inj} \} = 2 \mathcal{L} \left[ \int_0^t \frac{C_{loss}}{\sqrt{t - \lambda}} \frac{dA}{d\lambda} d\lambda \right] + (w_f + 2s_p) \mathcal{L} \left\{ \frac{dA(t)}{dt} \right\}$$  \hspace{1cm} (4-9)
Solving for $L\{A(t)\}$,

$$
L\{A(t)\} = \frac{q_{\text{inj}}}{s} \left[ \frac{1}{s^{3/2}} \frac{2C_{\text{loss}} \sqrt{\pi}}{(w_f + 2s_p)^{3/2}} \right] = \frac{q_{\text{inj}}}{(w_f + 2s_p)^{3/2} b^2} \left[ \frac{1}{s^{3/2}} \left( \frac{1}{s} + \frac{b}{s^{3/2}} \right) \right] (4-10)
$$

Where, $b = \frac{2C_{\text{loss}} \sqrt{\pi}}{(w_f + 2s_p)}$

The Laplace inversion solution yields:

$$
A(t) = \frac{q_{\text{inj}}}{(w_f + 2s_p)^{3/2} b^2} \left[ e^{b^2 \text{erfc}(b \sqrt{t})} - 1 + 2b \sqrt{\frac{t}{\pi}} \right] (4-11)
$$

Substituting $b = \frac{2C_{\text{loss}} \sqrt{\pi}}{(w_f + 2s_p)}$,

$$
A(t) = \frac{q_{\text{inj}}}{8\pi C_{\text{loss}}^2} \left[ e^{(2C_{\text{loss}} \sqrt{\pi} / w_f)^2} \text{erfc}\left( \frac{2C_{\text{loss}} \sqrt{\pi} t}{(w_f + 2s_p)} \right) + 4C_{\text{loss}} \sqrt{\pi} - 1 \right] (4-12)
$$

Rearranging,

$$
A(t) = \frac{q_{\text{inj}} (w_f + 2s_p)}{8\pi C_{\text{loss}}^2} \left[ e^{x^2 \text{erfc}(x)} + \frac{2x}{\sqrt{\pi}} - 1 \right] (4-13)
$$

Where, $x = \frac{2C_{\text{loss}} \sqrt{\pi} t}{w_f + 2s_p}$

Burger et al. (1985) approximation is used, $e^{x^2 \text{erfc}(x)} = \frac{1}{\sqrt{\pi} x}$ for $3 \leq x \leq 20$, leads to an engineering solution:

$$
\left( e^{x^2 \text{erfc}(x)} + \frac{2x}{\pi} - 1 \right) = \frac{x^2}{1 + 0.85x} (4-14)
$$
And,

\[ A(t) = \frac{q_{\text{inj}}(w_f + 2s_p)}{8\pi C_{\text{low}}^2} \left[ \frac{x^2}{1 + 0.85x} \right] \quad (4-15) \]

Using fracture surface area \( A(t) = Lh \), one obtains:

\[ L = \frac{5.6146 q_{\text{inj}}(w_f + 2s_p)}{8\pi h C_{\text{low}}^2} \left[ \frac{x^2}{1 + 0.85x} \right] \quad (4-16) \]

Fracture width and length are the unknowns in Equation 4-16.

In practice, the width of the fracture is assumed constant and replaced by the average fracture width \( \bar{w} \):

\[ \bar{w} = \frac{2}{3} w(0) \quad (4-17) \]

In this study, the hydraulic fracture length and width is solved simultaneously using a practical iterative process. The flowchart for calculating fracture length and width is presented in Figure 4-12.
4.2.2 Wishbone Example

The hydraulic fracture half length and width for eleven wells in Wishbone section is calculated. Then, each well’s SRV results obtained from rate-transient analysis in Chapter 3 are compared with the results obtained from fracture propagation model. The following is the workflow:

- Use PKN model to estimate hydraulic fracture length and width.
- Apply multiphase rate-transient analysis to identify the flow regimes.
- If the boundary-dominated flow is observed, calculate the SRV and hydraulic fracture half-length $y_f$. 

Figure 4-12 Flowchart for calculating fracture length and width
Input for this model are pumped fluid volume, pumped proppant amount, and the average pumping rate per stage. The example well completion data for the wells from Wishbone section used in this study is tabulated in Table 4-1.

Table 4-1 Example completion data

<table>
<thead>
<tr>
<th></th>
<th>Total Stage Fluid Volume (bbl)</th>
<th>Stage Prop Weight (lb)</th>
<th>Average pump rate (bpm)</th>
<th>Total Stage Fluid Volume (bbl)</th>
<th>Stage Prop Weight (lb)</th>
<th>Average pump rate (bpm)</th>
</tr>
</thead>
<tbody>
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<td>114,942</td>
<td>31.3</td>
<td>17</td>
<td>2,288</td>
<td>48,481</td>
</tr>
<tr>
<td>2</td>
<td>1,911</td>
<td>60,641</td>
<td>42.8</td>
<td>18</td>
<td>2,147</td>
<td>49,347</td>
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<tr>
<td>3</td>
<td>1,933</td>
<td>63,320</td>
<td>35.7</td>
<td>19</td>
<td>2,163</td>
<td>50,218</td>
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<tr>
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<tr>
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<td>1,901</td>
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<td>72,687</td>
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<td>22</td>
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<tr>
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<td>31</td>
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<tr>
<td>16</td>
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<td>71,117</td>
<td>60.2</td>
<td>32</td>
<td>2,777</td>
<td>82,663</td>
</tr>
</tbody>
</table>

In addition to the hydraulic fracture treatment data, measured mechanical properties from logs in Wattenberg Field (Young’s modulus and Poisson’s ratio), are used as an input to the model. Table 4-2 summarizes the rock properties used as an input parameter for the PKN model.

Table 4-2 Reservoir rock properties

<table>
<thead>
<tr>
<th>Young Modulus (E) (\times10^6\ \text{psi})</th>
<th>Poisson ratio (\nu)</th>
<th>Porosity (\phi)</th>
<th>Thickness of the reservoir (h) (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.00</td>
<td>0.26</td>
<td>0.10</td>
<td>80</td>
</tr>
</tbody>
</table>
The outputs of the model are hydraulic fracture half-length and width per stage. The estimated hydraulic fracture half-length \((y_f)\) and width \((w_f)\) for two of the wells in the Wishbone section are tabulated in Table 4-3.

<table>
<thead>
<tr>
<th>Well</th>
<th>Avg. proppant volume per stage</th>
<th>Avg. fluid injected per stage</th>
<th>Avg. Pumping rate per stage</th>
<th>Estimated (y_f) (ft)</th>
<th>Estimated (w_f) (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1N</td>
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<td>0.67</td>
<td>0.96</td>
<td>376</td>
<td>0.24</td>
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<tr>
<td>11N</td>
<td>1.39</td>
<td>1.33</td>
<td>1.04</td>
<td>564</td>
<td>0.27</td>
</tr>
</tbody>
</table>

### 4.3 Diagnostic Fracture Injection Testing (DFIT) in Unconventional Reservoirs

DFIT is the pressure transient testing for ultra-low unconventional reservoirs to determine reservoir and fracture properties in a very short period of time. It is also called minifrac, datafrac, mini fall-off, etc. but all refer to the same test sequence, i.e. injection followed by the fall-off tests. During the injection test, a relatively small volume (for example, 5-30 bbls for shale reservoirs and 50-100 bbls for unconventional tight sands) of fluid including typically 2-4 % KCl is injected into the formation at rates of 1-6 bpm without proppant to break down the formation and create a fracture. Then, this is followed by the fall-off test which refers to shut-in period for 3-14 days to allow the fracturing fluid leak-off to the formation and observe the closure of the fracture.

Main key parameters that can be obtained from a DFIT test are:

- Fracture closure pressure \((p_c)\) which is used for proppant selection and hydraulic fracture permeability calculation
- Efficiency of fluid which is used for hydraulic fracture geometry such width \((w_{HF})\) and length \((y_{HF})\) calculation
- Pressure capacity of the formation which is used for transmissibility \((kh/\mu)\) of the reservoir and fracture conductivity \((C_{FD} = \frac{k_{HF}w_{HF}}{k_{y_{HF}}})\) estimations
- Minimal stress contrast which is used for minimum and maximum horizontal stress estimation
- Formation leak-off mechanisms and loss coefficients which is used for maximum sand concentration estimation and pad volume requirements

4.3.1 DFIT Analysis

The analysis of DFIT data is performed in two parts which are pre-closure transient analysis (PCA) and after-closure transient analysis (ACA). There are basically three techniques used for both PCA and ACA analysis for after shut-in period: Nolte G-function, G-function log-log, square root of shut-in time. The radial flow in the reservoir is the identification of complete closure of the fracture.

The typical pressure behavior of DFIT (Figure 4-13) shows that after the fracture is created the pressure starts to decline rapidly and stabilizes. The instantaneous shut-in pressure (ISIP) is determined which is the first point where pump is shut-off.

Figure 4-13 DFIT test overview plot

The closure pressure is the state where the fracture is mechanically closed whereas it is still hydraulically connected to the reservoir. The pressure eventually approaches to the reservoir pressure after the fracture is closed mechanically and the pressure decline is only controlled by the reservoir. The fracture closure pressure is determined using the Nolte G-function plot, which is the dimensionless time function, was introduced by Nolte, 1979. The G-function, which allows a linear
relationship between pressure and leak-off coefficient, is designed to linearize the pressure behavior (Eq. 4-18) during normal leak-off (Economides and Nolte, 1987):

\[ p_{ws} - p_w(\Delta t_D) = \frac{\pi r_p C_L \sqrt{t_p}}{2c_f} G(\Delta t_D) \]  

(4-18)

And,

\[ G(\Delta t_D) = \frac{4}{\pi} \left[ g(\Delta t_D) - g_0 \right] \]  

(4-19)

\[ \Delta t_D = \frac{\Delta t}{t_p} \]  

(4-20)

Where,  \( p_{ws} \) is the bottomhole shut-in pressure,  \( p_w \) the bottomhole flowing pressure at the start of shut-in,  \( r_p \) the ratio of permeable area to fracture area,  \( c_f \) fracture compliance,  \( C_L \) fluid leak-off coefficient.

The deviation from semi-log derivative of pressure with respect to G-function \( \left( G \frac{dP}{dG} \right) \) versus G-function on a Cartesian plot through the origin indicates the closure pressure,  \( p_c \) (Barree et al., 2009). The details of choosing the correct closure pressure is explained by Barree et al., 2014 in detail.

Pre-closure analysis is performed for the leak-off area where fracture and reservoir are still in contact. Thus, generally the flow regime is dominated by linear flow during PCA. At the moment of fracture closure the leak-off area is changed from fracture- reservoir contact to wellbore/reservoir contact. Hence, the flow is dominated by the radial flow regime.

The log-log analysis of pressure and derivative of pressure versus time is plotted to identify the flow regimes. The linear flow during PCA exhibits a positive \( \frac{1}{2} \) slope on the log-log plot whereas during ACA exhibits a negative \( \frac{1}{2} \) slope (Barree et al., 2014). The pseudo-radial flow exhibits a slope of negative 1 during ACA. The summary of slope characteristics on log-log plot is identified for each flow regime in Table 4-4 (Barree et al., 2009)
Table 4-4 Log-log graph characteristic slopes

<table>
<thead>
<tr>
<th>Log-Log Graph</th>
<th>Before Closure</th>
<th>After Closure</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta p_{w} ) vs. ( t )</td>
<td>1/4</td>
<td>1/2</td>
</tr>
<tr>
<td>( \Delta p_{w} ) vs. ( t_{a} )</td>
<td>( -3/4 )</td>
<td>( -1/2 )</td>
</tr>
<tr>
<td>( \partial \Delta p_{w} / \partial t ) vs. ( t )</td>
<td>1/4</td>
<td>1/2</td>
</tr>
<tr>
<td>( \partial \Delta p_{w} / \partial t_{a} ) vs. ( t_{a} )</td>
<td>5/4</td>
<td>3/2</td>
</tr>
</tbody>
</table>

The linear flow analysis is performed by plotting square root of time, \( \sqrt{t} \) versus pressure \( p_{w} \) and derivative of pressure. The spurt, treatment fluid efficiency, hydraulic fracture geometry, and fracture closure pressure are obtained by the linear flow analysis (Economides and Nolte 2000). The transmissibility of the reservoir (\( kh/\mu \)) is obtained by the pseudo-radial flow analysis (Barree et al., 2009).

4.3.2 DFIT Analysis in Wishbone

Four DFIT was conducted in vertical offset wells from surrounding Wishbone area. The three of the DFIT were conducted in the Niobrara formation and one of them was conducted in Codell formation. The results of linear flow and radial flow analysis are tabulated in Table 4-5. The identification of pressure dependent leak-off (PDL) and transverse storage leak-off types indicate the existence of secondary fractures in Niobrara formation.
Table 4-5 Results of DFIT Analysis

<table>
<thead>
<tr>
<th></th>
<th>Gobbler Well 1</th>
<th>Gobbler Well 2</th>
<th>Gee Well 1</th>
<th>Gobbler Well 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Formation</td>
<td>Codell</td>
<td>Niobrana</td>
<td>Niobrana B Chalk</td>
<td>Niob Shale</td>
</tr>
<tr>
<td>Depth (ft TVD)</td>
<td>7543 ft TVD</td>
<td>7300 ft TVD</td>
<td>7172 ft TVD</td>
<td>7361 ft TVD</td>
</tr>
<tr>
<td>BH Closure (psi)</td>
<td>5272</td>
<td>5434</td>
<td>4991 psi</td>
<td>5277 psi</td>
</tr>
<tr>
<td>BH Closure Gradient (psi/ft)</td>
<td>0.7 psi/ft</td>
<td>0.74 psi/ft</td>
<td>0.7 psi/ft</td>
<td>0.72 psi/ft</td>
</tr>
<tr>
<td>BH ISIP (psi)</td>
<td>6162</td>
<td>6045</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BH FRAC Gradient - ISIP (psi/ft)</td>
<td>0.82 psi/ft</td>
<td>0.83 psi/ft</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reservoir Pressure (psi)</td>
<td>4512 psi</td>
<td>4527 psi</td>
<td>4090 psi</td>
<td>4460 psi</td>
</tr>
<tr>
<td>Reservoir Pressure Gradient (psi/ft)</td>
<td>0.6 psi/ft</td>
<td>0.62 psi/ft</td>
<td>0.57 psi/ft</td>
<td>0.60 psi/ft</td>
</tr>
<tr>
<td>Pore pressure from</td>
<td>Linear flow analysis</td>
<td>Linear flow analysis</td>
<td>Linear flow analysis</td>
<td>Linear flow analysis</td>
</tr>
<tr>
<td>Leakoff Type</td>
<td>Normal Matrix</td>
<td>Transverse Storage</td>
<td>Pressure Dependent</td>
<td></td>
</tr>
<tr>
<td>Permeability (mD)</td>
<td>0.0552 mD</td>
<td>0.0183 mD</td>
<td>0.007 mD</td>
<td>0.004 mD</td>
</tr>
<tr>
<td>Flow Capacity (md-ft)</td>
<td>0.55 md-ft</td>
<td>0.73 md-ft</td>
<td>0.161 md-ft</td>
<td>0.106 md-ft</td>
</tr>
<tr>
<td>Process Zone Stress (PZS = ISIP - Pc)</td>
<td>890</td>
<td>611</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
CHAPTER 5
NUMERICAL MODELING

The stress-dependent deformation in hydrocarbon bearing shale formations affects the production decline trends. Therefore, determining the transport mechanisms that affect well performance in liquids-rich unconventional reservoirs is critical because of the ever-changing nanoscale flow and transport behavior in shale reservoirs. In this chapter, to account for the complexity, which incorporates a bimodal porosity and permeability distribution for the rock matrix and added a set of macro-fractures, in the stimulated reservoir volume (SRV) a novel dual-porosity fully-coupled geomechanics and flow model is presented to understand the effect of rock deformation on production in unconventional reservoirs. Production data from Niobrara formation will be presented to demonstrate the efficacy of our model in Chapter 6.

This chapter includes (1) dual-porosity multi-phase flow formulation using poroelasticity theory, (2) mathematical formulation, and (3) model verification.

5.1 Dual-Porosity Multi-Phase Flow Model Using Poroelasticity Theory

Production performance of low-permeability unconventional reservoirs heavily relies on the presence of micro and macro fractures. Changes in reservoir pore pressure and temperature during injection or production affect rock deformation, which, in turn, causes alteration of porosity and permeability. During production, reduction in pore pressure causes a change in the state of stress in rock frame. The change in the state of stress is often reported as the net stress or effective stress because of the direct relation between formation permeability, porosity, and formation subsidence. To demonstrate these effects, I have developed a mathematical model to quantify and assess the viability of the effects of rock deformation on production.

5.1.1 Literature Review

The concept of effective stress dates back to the experimental work performed by Terzaghi (1925), who defined the concept when he was studying the consolidation of soils under a constant load without lateral movement – the uniaxial strain. Later, Biot (1941), developed the
mathematical framework for the poroelasticity theory by extending Terzaghi’s uniaxial strain to the three-dimensional space. In poroelasticity theory, the fluids completely fill the pores of the elastically deformable porous media, and the transport of pore fluids obeys Darcy law (Bundschuh, 2010). This concept of poroelasticity can be extended to the dual-porosity media (that is, matrix-fracture systems). In fact, several idealizations exist for the matrix-fracture media such as the dual-porosity and dual-permeability concepts. In the dual-porosity systems, fractures form a continuum of interconnected fracture channels while the matrix is a porous reservoir rock surrounded by the fractures. The majority of the dual-porosity models in deformable rocks pertain to the single-phase flow (Berryman and Wang, 1995; Khalili and Valliappan, 1996), while in the petroleum reservoirs the multi-phase flow of water, oil, and gas must be considered. Thus, the formulation must be extended to multi-phase flow.

To use the Biot poroelasticity theory with the multiphase flow modeling, partial saturation of porous media by different fluids must be considered. Biot theory (Biot, 1941), assumes that the porous rock is fully saturated by only one fluid. In this work, our system is assumed as a single composite fluid having an averaged total fluid bulk modulus using Wood equation (Wood, 1941) so the Biot model of a single fluid phase can be applied to multiphase systems. In addition to the fluid inside the rock, solid grains and rock skeleton contribute to the total strength of the combined system. The relative stiffness of rock skeleton, solid grains, and pore fluid could be identified by different bulk moduli values which are also used to define Biot moduli for both fracture and matrix.

There are several attempts to numerically simulate multi-phase flow in deforming rocks using either single-porosity or dual-porosity systems. The earliest application of poroelastic theory to dual-porosity systems was reported by (Aifantis, 1977, and Aifantis, 1980). Another dual-porosity poroelastic model for single-phase flow was provided by Bai (1999), who demonstrated the significance of the fracture deformation which was larger than the matrix deformation due to the compressibility difference between fracture and matrix. Lewis et al. (1997) extended the dual-porosity deformable model to three-phase flow without including fracture deformation and cross-coupling terms. Several years later, the fracture deformation and a cross-coupling to the matrix blocks were included in multiphase isothermal models by Pao et al. (2002) and Lewis et al. (2002). Bagheri and Settari (2008) used an equivalent single-porosity model for rock deformation.
Similarly, Kim et al. (2013) studied multiphase flow coupled to geomechanics for a single-porosity model.

5.2 Mathematical Formulation

To model multiple continua, consisting of fracture and matrix of different sizes dual-porosity models are generally used. The dual-porosity model built in this study is based on the assumption that matrices feed the micro/macro fractures created in the SRV, and eventually the fractures are connected to the hydraulic fracture and the horizontal wellbore.

The governing equations for mass flow for multi-phase and the rock deformation using linear poroelasticity in dual-porosity media will be presented. The assumptions for the model development are:

- The system consists of three phase flow that are water, oil, and gas;
- The reservoir is isothermal and homogenous, with constant thickness;
- Fluid flow is under advective flow which consists of viscous, and capillary forces where gravity effects are ignored;
- The change in volumetric strain with accompanying changes in porosity and permeability both in fracture and matrix with respect to change in stress is included;
- For the uniaxial strain case (commonly used in practice): $\varepsilon_x = \varepsilon_y = 0$

5.2.1 Mass Transport Including Bulk Rock Deformation

In formulating the mass transport equations, it is assumed that the saturated porous media consists of two mobile phases—a solid phase and a fluid phase which can flow as a result of appropriate flow potential gradients. The mass balance equations in the stationary (or Eulerian) coordinate for the solid phase and fluid phase at a fixed point in space are:

$$-\nabla \cdot (\rho_s (1-\phi) u_s) = \frac{\partial}{\partial t} [\rho_s (1-\phi)]$$

(5-1)

$$-\nabla \cdot (\rho_f \phi u_j) = \frac{\partial}{\partial t} (\rho_f \phi)$$

(5-2)
\[ \tilde{u}_f = \frac{\vec{r}}{\phi} \]  \hspace{1cm} (5-3)

Where, \( \rho_s \) is the solid density, \( \phi \) porosity, \( \vec{r} \) the interstitial solid phase velocity, \( \rho_f \) fluid density, \( \vec{u}_s \) interstitial fluid phase velocity, and \( \vec{v}_f \) Darcy fluid velocity. The Darcy velocity is measured with respect to the solid phase which flows with its own velocity. Thus, Darcy velocity takes the following form in the moving (or Lagrangian) coordinates:

\[ -\frac{k}{\mu} \nabla p = \phi (\vec{r} - \vec{r}) \]  \hspace{1cm} (5-4)

Where,

\[ \vec{r} = \left( \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \right) \]  \hspace{1cm} (5-5)

After rearranging the above equation, we obtain

\[ \vec{r} \]  \hspace{1cm} (5-6)

Substituting the above equation in the mass balance equation for the fluid phase, we obtain:

\[ \nabla \cdot \left( \rho_f \frac{k}{\mu} \nabla p \right) - \nabla \cdot \left( \rho_f \rho \frac{\vec{r}}{\phi} \right) = \frac{\partial}{\partial t} \left( \rho_f \phi \right) \]  \hspace{1cm} (5-7)

After further manipulations, one would obtain the following equivalent equations:

\[ \nabla \cdot \left( \rho_f \frac{k}{\mu} \nabla p \right) = \rho_f \phi \left[ \nabla \cdot \vec{r} + \left( \frac{1}{\rho_f} \frac{d\rho_f}{dt} + \frac{1}{\phi} \frac{d\phi}{dt} \right) \right] \]  \hspace{1cm} (5-8)

\[ \nabla \cdot \left( \rho_f \frac{k}{\mu} \nabla p \right) = \rho_f \phi \left[ \frac{1}{V_b} \frac{dV_b}{dt} + \frac{1}{\rho_f} \frac{d\rho_f}{dt} + \frac{1}{\phi} \frac{d\phi}{dt} \right] \]  \hspace{1cm} (5-9)
Where, $V_b$ is bulk volume

\[
\nabla \cdot \left( \frac{k}{\mu} \rho_f \nabla p \right) = \rho_f \phi \left\{ \frac{1}{V_b} \frac{dV_p}{dt} + \phi \frac{1}{\rho_f} \frac{d\rho_f}{dt} \right\}
\]

(5-10)

Or,

\[
\nabla \cdot \left( \frac{k}{\mu} \rho_f \nabla p \right) = \rho_f \phi \left\{ \frac{1}{V_b} \frac{dV_p}{dt} + \phi \frac{1}{\rho_f} \frac{d\rho_f}{dt} \right\}
\]

(5-11)

Where, $V_p$ is pore volume

Using Jaeger et al. (2007) notation, the bracketed expression in Equation 5-11 is noted as

\[
\frac{d\xi}{dt}
\]

which is the time derivative of the volumetric change in the fluid content that is solely due to mass transfer:

\[
\frac{d\xi}{dt} = \frac{1}{V_b} \frac{dV_p}{dt} + \phi \frac{1}{\rho_f} \frac{d\rho_f}{dt}
\]

(5-12)

Jaeger et al. (2007) show that:

\[
\nabla \cdot \left( \frac{\rho_f}{\mu} \nabla p \right) = \rho_f \left\{ \frac{1}{M} \frac{\partial p}{\partial t} - \alpha \frac{\partial \varepsilon_v}{\partial t} \right\}
\]

(5-13)

Where,
\[
\frac{1}{M} = \frac{\phi}{K_\beta} + \frac{\alpha - \phi}{K_b}
\]  
(5-14)

\[
\alpha = 1 - \frac{K_b}{K_s}
\]  
(5-15)

\[
\kappa_b = \kappa_{frame}
\]  
(5-16)

In the above equations, \(M\) is the Biot modulus, \(K_\beta\) fluid modulus, \(\alpha\) Biot coefficient, \(K_b\) bulk or frame modulus, \(K_s\) solid modulus, \(\mu\) viscosity of the fluid, \(k\) formation permeability. The Equation 5-13 takes into account the linear elastic deformation of the rock frame and its fluid content as a result of pore pressure and net stress change.

The representative elementary volume (REV) of the dual-porosity systems used in this model consists of the solid material, the porous matrix block material without fractures and the reservoir rock frame containing of solids, matrix, and fractures (Figure 5-1).

![Figure 5-1: Matrix block without fractures (a) with interconnected fractures (b)](image)

To extend the single-phase model (Equation 5-13) to the dual-porosity model, the following set of equations are used in such a way to properly couple the fluid flow equations in both fracture and matrix. Specifically, I defined an appropriate matrix and fracture Biot modulus for the dual-porosity reservoirs. The matrix modulus is the commonly measured rock modulus
obtained from the laboratory experiments. However, the fracture modulus is the bulk modulus of the fractured rock.

For clarity, I first present the single-phase dual-porosity model. Then, this model will be extended to multi-phase flow. The single-phase, dual-porosity model is:

\[
\nabla \cdot \left[ \frac{k_{f,\text{eff}}}{\mu} \left( \nabla p_f - \gamma_f \nabla D \right) \right] - \tau + \hat{q} = \frac{1}{M_f} \frac{\partial p_f}{\partial t} - \alpha_f \frac{\partial \varepsilon_f}{\partial t}
\]

(5-17)

Where,

\[
\tau = \sigma \left( \frac{k_m}{\mu} \right) \left( p_f - p_m \right)
\]

(5-18)

\[
k_{f,\text{eff}} = k_f \phi_f + k_m ; \quad k_f \phi_f
\]

(5-19)

Here, \(k_{f,\text{eff}}\) is the effective permeability of the formation in the dual-porosity systems, \(k_f\) micro/macro fracture permeability embedded in the matrix, \(\phi_f\) fracture porosity, \(\mu\) the viscosity, \(p_f\) pressure in the fracture, \(\gamma_f\) the specific gravity of the fluid, \(D\) the depth, \(\tau\) the transfer function between matrix and fracture, \(\dot{q}\) the rate per unit rock volume, \(M_f\) Biot modulus for the fracture, \(\alpha_f\) Biot coefficient for the fracture medium, \(\sigma\) the shape factor, \(k_m\) the matrix permeability, \(p_m\) the pressure in the matrix.

Modeling rock deformation in the dual-porosity system requires a practical description of bulk moduli. Moreover, considering fractures are the most compressible and the solid minerals are the least compressible medium in the dual-porosity system, the magnitude of each bulk modulus can vary from the highest to lowest as the solid grain, the matrix block without fractures, and the matrix block with fractures. Therefore, the inverse of Biot modulus for the fracture medium (Equation 5-20) is defined as a function of the fracture fluid modulus and the rock frame modulus with the presence of fractures. The latter is an extension of the single-porosity definition given by Fjær et al. (2008).
\[
\frac{1}{M_f} = \frac{\phi_f}{K_{fl}} + \left( \frac{\alpha_f - \phi_f}{K_{dfb}} \right) \tag{5-20}
\]

\[
\alpha_f = 1 - \frac{K_{dfb}}{K_{dm}} \tag{5-21}
\]

Where, \( M_f \) is the Biot modulus of the fracture medium, \( K_{fl} \) the bulk modulus of the fluid (inverse of the fluid compressibility), \( \alpha_f \) Biot coefficient for the fracture, \( K_{dfb} \) the bulk drained modulus of matrix blocks containing fractures, and \( K_{dm} \) the bulk drained modulus of matrix blocks without fractures.

Similarly, the coupled mass balance equation for the matrix is:

\[
\tau = \frac{1}{M_m} \frac{\partial p_m}{\partial t} - \alpha_m \frac{\partial \varepsilon_m}{\partial t} \tag{5-22}
\]

And, the inverse of Biot modulus for the matrix medium is:

\[
\frac{1}{M_m} = \frac{\phi_m}{K_{fl}} + \left( \frac{\alpha_m - \phi_m}{K_{dm}} \right) \tag{5-23}
\]

\[
\alpha_m = \frac{K_{dfb}}{K_{dm}} - \frac{K_{dfb}}{K_{sm}} \tag{5-24}
\]

Where, \( M_m \) is the Biot modulus of the matrix, \( \alpha_m \) Biot coefficient for the matrix, and \( K_{sm} \) the bulk modulus of solid minerals in the matrix.

The Biot coefficient of the total system is given below:

\[
\alpha_m + \alpha_f = 1 - \frac{K_{dfb}}{K_{sm}} \tag{5-25}
\]

The approach for extending single-phase Biot formulation to multiphase fluid could be explained with a statement from Toms et al. (2006) when multiphase fluids are distributed on a fine scale, and the interaction occurs between the rock and composite fluids, they can be regarded.
as a single composite fluid whose compressibility could be calculated by the average of its constituent compressibilities (Toms et al., 2006). The Biot moduli calculations presented in Equation 5-20 and Equation 5-23, and our multiphase diffusivity equations are based on this aforementioned physical representation of immiscible fluids in the micro-scale pores and fractures.

Using the definition of increment of total fluid volumetric content in linear poroelasticity theorem (Equation 5-13), the global pressure equation in the fracture for the uncoupled multiphase dual-porosity system takes the final form for coupled multi-phase mass balance equation in the fracture in Equation 5-26.

$$
\left\{ \nabla \left[ k_{\text{eff}} \left( \lambda_{\text{wf}} + \lambda_{\text{of}} + \lambda_{\text{of}} \right) \nabla p_{\text{of}} - \lambda_{\text{wf}} \nabla p_{\text{wf}} + \lambda_{\text{of}} \nabla p_{\text{of}} \right] \right\} - \left( \tau_{w} + \tau_{o} + \tau_{g} \right) + \left( B_{\text{wf}} \dot{q}_{\text{wf}} + B_{\text{of}} \dot{q}_{\text{of}} + B_{\text{of}} \dot{q}_{\text{of}} \right) = \frac{1}{M_{f}} \frac{\partial p_{\text{of}}}{\partial t} + \frac{\alpha_{f}}{\left( \lambda + 2G \right)} \frac{\partial p_{\text{of}}}{\partial t}
$$

Where, $\lambda_{\text{wf}}, \lambda_{\text{of}}, \text{and } \lambda_{\text{of}}$ are the mobilities of water, oil, and gas in the fracture respectively, $p_{\text{of}}$ the pressure of oil in the fracture, $p_{\text{wf}}$, $p_{\text{of}}$ the water-oil capillary pressure and gas-oil capillary pressure in the fracture, $B_{\text{wf}}, B_{\text{of}}, B_{\text{of}}$ formation volume factor for water, oil, and gas in the fracture, $\dot{q}_{\text{wf}}, \dot{q}_{\text{of}}, \dot{q}_{\text{of}}$ rate of water, oil, and gas per unit rock volume, $\lambda$ Lame coefficient, and $G$ shear modulus

Similarly, the coupled global pressure equation in the matrix takes the form:

$$
\tau_{w} + \tau_{o} + \tau_{g} = \frac{1}{M_{m}} \frac{\partial p_{\text{om}}}{\partial t} + \frac{\alpha_{m}}{\lambda + 2G} \frac{\partial p_{\text{om}}}{\partial t}
$$

To solve the saturation equations for water and gas in fracture medium, Equation 5-28 and Equation 5-29 are used respectively.

$$
\nabla \cdot \left[ \frac{k_{\text{off}} \lambda_{\text{of}}}{B_{\text{of}}} \left( \nabla p_{\text{of}} - \gamma_{\text{of}} \nabla D - \nabla p_{\text{cotf}} \right) \right] - \tau_{w} + \dot{q}_{\text{of}} = \frac{\phi_{f}}{B_{\text{of}}} \left[ s_{\text{of}} \left( \epsilon_{\text{of}} + \hat{\epsilon}_{\text{of}} \right) \frac{\partial p_{\text{of}}}{\partial t} + \frac{\partial}{\partial t} \left( S_{\text{of}} \right) \right]
$$

71
\[
\begin{aligned}
\left[ \nabla \cdot \left[ k_{f,\text{eff}} \hat{\lambda}_{\text{eff}} \left( \nabla p_{\text{eff}} - \gamma_{\text{sf}} \nabla D + \nabla p_{\text{swf}} \right) \right] - \tau_s + B_{sf} \hat{q}_{sf} \right] &= \phi_f \frac{\partial S_{sf}}{\partial t} + \phi_f \left[ S_{sf} \hat{c}_{sf} + S_{sf} \left( \frac{B_{sf}}{B_{sf}} \frac{\partial R_{\text{swf}}}{\partial p_{sf}} \right) \right] \\
&+ S_{sf} \left( \frac{B_{sf}}{B_{sf}} \frac{\partial R_{\text{swf}}}{\partial p_{sf}} \right) + S_{sf} \hat{c}_{sf} \\
&= \phi_m \left[ S_{wm} \left( c_{wm} + \hat{c}_{wm} \right) \frac{\partial p_{\text{wm}}}{\partial t} + \frac{\partial}{\partial t} (S_{wm}) \right] \quad (5-29)
\end{aligned}
\]

Where, \( c_{sf} \) is the fracture compressibility, \( \hat{c}_{sf} \) compressibility of water and gas in the fracture respectively, \( S_{sf} \) saturation of water in the fracture, \( S_{sf} \) saturation of gas in the fracture, \( R_{\text{swf}}, R_{\text{swf}} \) solution gas-oil ratio and gas-water ratio in the fracture.

Similarly, Equation 5-30 and Equation 5-31 are used to solve for water and gas saturation in the matrix respectively:

\[
\sigma k_{m} \hat{\lambda}_{\text{m,eff}} \left[ \left( p_{sf} - p_{\text{wm}} \right) + S_{wm} \left( c_{wm} + \hat{c}_{wm} \right) \frac{\partial p_{\text{wm}}}{\partial t} + \frac{\partial}{\partial t} (S_{wm}) \right] \quad (5-30)
\]

Where, \( h_{sf}, h_{wm} \) represent the height of the water column in fracture and matrix respectively, and \( h_{sf}, h_{wm} \) the height of the gas column in fracture and matrix.

**5.2.2 Rock Deformation**

In linear elastic material, rocks expand or contract linearly with applied stress. After applying force balance on a representative elementary volume, we obtain the Navier equation or equation of motion of rock deformation:

\[
\nabla \cdot \sigma + \rho \ddot{F} = \rho \frac{\partial^2 \hat{u}}{\partial t^2} \quad (5-32)
\]
Where \( \mathbf{u} = [u \ v \ w]^T \), the displacement vector for rock particles.

When, particle velocity is very small, equation of motion will have the following form:

\[
\nabla \cdot \sigma + \rho \mathbf{F} = 0
\]

\( (\lambda + G)\nabla (\nabla \cdot \mathbf{u}) + G \nabla^2 \mathbf{u} + \rho \mathbf{F} = 0 \) \hfill (5-34)

For uniaxial strain problem in the vertical direction, the particle displacement in the vertical direction follows the following differential equation (Eq. 5-35).

\[
(\lambda + 2G) \left( \frac{\partial^2 \mathbf{w}}{\partial z^2} \right) = -(\alpha_f + \alpha_m) \frac{\partial \mathbf{p}}{\partial z}
\]

\hfill (5-35)

**5.2.3 Porosity and Permeability Change with Time**

The governing equations for fluid flow and rock deformations are solved numerically using a stationary coordinate frame which is based on the time-independent Eulerian description. Thus, the true porosity (Equation 5-36) calculated from the rock deformation model cannot be directly used and should be replaced by reservoir simulation porosity based on the stationary computing grid (Eq. 5-37) in the model formulation (Thomas et al., 2003).

\[
\phi_{\text{true,}i} = 1 - (1 - \phi_{\text{init},i}^{\text{true}}) e^{\alpha_i \varepsilon_i} \quad ; i = \text{matrix, fracture}
\]

\hfill (5-36)

\[
\phi_{\text{sim,}i} = \left[ \phi_{\text{init},i}^{\text{sim}} + (e^{-\alpha_i \varepsilon_i} - 1) \right] \quad ; i = \text{matrix, fracture}
\]

\hfill (5-37)

Where, \( \phi_{\text{init},i}^{\text{true}} \) represents the initial porosity of matrix and fracture.

Generally, porosity should be calculated using Equation 5-38 if there is no bulk rock deformation in the model formulations; however, for convenience, the effective mean stress is replaced by pore pressure.
\[ \phi_{sim,i} = \phi_{true,i} = \phi_i^{init} e^{-c_{\phi,i}(\sigma_{mean,i} - \sigma_{mean,init})} \]  
(5-38)

Nonetheless, the required compressibilities used in Equation 5-38 may be estimated using Equation 5-39.

\[ c_{\phi,f} = \frac{1}{K_{dfb}} \quad \text{and} \quad c_{\phi,m} = \frac{1}{K_{dm}} \]  
(5-39)

It should be noted that Equation 5-40 provided the pore compressibilities used in our linear elastic model.

\[ c_{\phi,f} = \frac{1}{\phi_f} \left( \frac{\alpha_f - \phi_f}{K_{dfb}} \right) \quad \text{and} \quad c_{\phi,m} = \frac{1}{\phi_m} \left( \frac{\alpha_m - \phi_m}{K_{dm}} \right) \]  
(5-40)

To update permeability which uses the relation between porosity and permeability the following equation is used (Jones, 1975):

\[ \left( \frac{\phi}{\phi^{init}} \right)^3 = \frac{k}{k^{init}} \]  
(5-41)

The fracture porosity, fracture permeability, effective reservoir permeability, and shape factor are all interconnected terms through the width of the fractures and the matrix block dimensions. Therefore, following equations (Equation 5-42, 5-43, and 5-44) should be carefully used in the models to work in a consistent system (Kazemi et al., 1993). Fracture porosity \( \phi_f \) remains essentially the same when the fracture surfaces move parallel to each other (slippage) because of the shear force. However, the fracture permeability \( k_f \) could decrease substantially because of the constriction (pore throat) effect because of the slippage. Therefore, a fracture roughness measure \( C \) is used in calculating the fracture permeability \( k_f \). Here \( C \) being equal to one indicates that there is no slippage, whereas if there is slippage \( C \) must be higher than one.
\[ \phi_j = w_j \left( \frac{1}{L_x} + \frac{1}{L_y} + \frac{1}{L_z} \right) \]  

(5-42)

\[ k_j = \frac{1}{C} \frac{w_j^2}{12} \]  

(5-43)

\[ \sigma = 4 \left( \frac{1}{L_x^2} + \frac{1}{L_y^2} + \frac{1}{L_z^2} \right) \]  

(5-44)

Where, \( w_j \) is the width of the fracture, \( L_x, L_y, L_z \) the dimensions of the matrix block, and \( C \) fracture roughness measure and diagenetic factor.

5.3 Numerical Discretization

Finite element method is generally preferred as a numerical formulation to solve partial differential equation (PDE) for rock deformation since it is considered as a better approach for modeling the deformation of complex isolated objects. However, in geosciences, implementation of finite difference for rock deformation is as powerful as finite element method due to the absence of pre-defined material interfaces (Gerya, 2010). Therefore, in this study to solve the (PDE) for both fluid flow and rock deformation, the finite difference technique is used. A block-centered grid and edge-centered grid were used for the mass transport and rock deformation control volumes respectively. The details of the finite difference implementation of the mathematical model can be found in Appendix-A.

5.4 Solution Algorithm

For the efficiency of the computation, we solved the global pressure equations for fracture and matrix, and the rock deformation simultaneously, porosity and permeability updates and saturations explicitly. The solution procedure is illustrated in Figure 5-2:
Figure 5-2 Flowchart of the coupled analysis single-phase (a) and multi-phase flow (b)

The primary variables and the corresponding equation is tabulated in Table 5-1:

<table>
<thead>
<tr>
<th>Primary Variables</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{of}^{n+1}$</td>
<td>Eq.5–26</td>
</tr>
<tr>
<td>$p_{on}^{n+1}$</td>
<td>Eq.5–27</td>
</tr>
<tr>
<td>$w_{m}^{n+1}$</td>
<td>Eq.5–35</td>
</tr>
<tr>
<td>$S_{wf}^{n+1}$, $S_{gf}^{n+1}$, $S_{of}^{n+1}$</td>
<td>Eq 5–28, Eq 5–30</td>
</tr>
<tr>
<td>$S_{wm}^{n+1}$, $S_{gm}^{n+1}$, $S_{om}^{n+1}$</td>
<td>Eq 5–29, Eq 5–31</td>
</tr>
</tbody>
</table>
5.5 Model Verification

To verify our models, the rate-transient analysis was used with the generated production data to back-calculate the effective permeability, which was input parameter in the numerical model. For fluid modeling in unconventional reservoirs, the accurate representation of the system consists of fractures and matrices. The flow hierarchy assumed in the model is that matrices feed micro and macrofractures locally, micro, and macrofractures flow in a linear fashion toward the hydraulic fracture, and subsequently to the horizontal well (Figure 5-3).

![Model representation with dual-porosity flow concept](image)

Figure 5-3 Model representation with dual-porosity flow concept

Initially, diagnostic plot (log-log) was used to identify the flow regimes (Figure 5-4 for single-phase flow, and Figure 5-5 for multi-phase flow), followed by applying linear flow analysis to calculate the effective permeability of the system (Kazemi et al., 2015).

The analyses of data from Figure 5-4 indicated that the effective permeability calculated from the 2-D, fully-coupled, single-phase, dual-porosity numerical model compares favorably with the analytical model results within 1.82% accuracy.
Similar verification was done for the 2-D, fully-coupled, multi-phase, dual-porosity model. The diagnostic plot and the linear flow analysis was performed (Figure 5-5), and the effective permeability back-calculated from the numerical model showed consistent results (error within 9%) with the analytical solutions despite the fact that the compressibility and porosity values are not constant.

![Figure 5-4 Single-phase log-log diagnostic plot (a) and linear flow analysis plot (b)](image)

![Figure 5-5 Multiphase-phase log-log diagnostic plot (a) and linear flow analysis plot (b)](image)
CHAPTER 6

NUMERICAL SIMULATION RESULTS

In this chapter, a dual-porosity fully-coupled geomechanics and flow model was developed to understand the effect of rock deformation on production in unconventional reservoirs. Production data from Niobrara formation will be presented to demonstrate the efficacy of our model.

This chapter includes (1) field application, (2) sensitivity analysis, and (3) discussion.

6.1 Field Application

6.1.1 Model Construction

6.1.1.1 Grid Construction

The stimulated reservoir volume for a single stage was constructed to simulate the production. The system was composed of a very high permeability hydraulic fracture, micro/macro fractures, and a matrix to represent dual-porosity media in the reservoir.

Logarithmic gridding algorithm (Appendix -B) was used to create a very fine gridding sizes at the vicinity of the hydraulic fracture and gradually increased as the grids were located away from the hydraulic fracture (Figure 6-1).

![Linear Flow towards hydraulic fracture](image)

Figure 6-1 Microfracture-Matrix system feeding a single HF

The simulation was run for a 2D model constructed using the reservoir dimensions tabulated in Table 1.
Table 6-1 Reservoir Dimensions

<table>
<thead>
<tr>
<th>$N_x, N_y, N_z$</th>
<th>7x1x7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta x$</td>
<td>Logarithmic</td>
</tr>
<tr>
<td>$\Delta y = y_{hf}$</td>
<td>440</td>
</tr>
<tr>
<td>$\Delta z$</td>
<td>12</td>
</tr>
<tr>
<td>$w_f$</td>
<td>0.25</td>
</tr>
<tr>
<td>$h$</td>
<td>84</td>
</tr>
<tr>
<td>$x_e$</td>
<td>60</td>
</tr>
</tbody>
</table>

### 6.1.1.2 Rock Properties

The SRV consisted of matrix and micro/macro fractures. The properties of matrix and fractures in the dual-porosity model used in the simulation run was tabulated in Table 6-2. The properties of matrix were obtained from MICP results (Chapter 2). The micro/macro fracture properties were calculated using Equation 6-1 and the results obtained from multi-phase RTA (Chapter 3).

The fracture permeability, $k_f$, can be calculated using the effective permeability, $k_{f,\text{eff}}$, definition:

$$k_{f,\text{eff}} = k_f \phi_f + k_m$$  \hspace{1cm} (6-1)

Where,

$k_{f,\text{eff}}$ is the effective permeability, $k_f$ fracture permeability, $k_m$ matrix permeability, $\phi_f$ fracture porosity

The fracture porosity in terms of width of the fractures and fracture spacing, shape factor defined by Kazemi (1987), and fracture permeability accounting for the fracture roughness measure, C, (Kazemi et al., 1993) are defined in Equation 6-2 to 6-4:

$$\phi_f = w_f \left( \frac{1}{L_x} + \frac{1}{L_y} + \frac{1}{L_z} \right)$$  \hspace{1cm} (6-2)
\[ k_f = \frac{1}{C} \frac{w_f^2}{12} \]  

(6-3)

\[ \sigma = 4 \left( \frac{1}{L_x^2} + \frac{1}{L_y^2} + \frac{1}{L_z^2} \right) \]  

(6-4)

Where, \( w_f \) is the width of the fracture, \( L_x, L_y, L_z \) the dimensions of the matrix block, and \( C \) fracture roughness measure and diagenetic factor.

\begin{table}
\centering
\begin{tabular}{|l|c|l|}
\hline
Matrix Porosity, \( \phi_m \) & 0.10 & fraction \\
\hline
Matrix Permeability, \( k_m \) & 3.14E-6 & mD \\
\hline
Effective permeability, \( k_{f,\text{eff}} \) & 0.003 & mD \\
\hline
Fracture porosity, \( \phi_f \) & 0.0025 & fraction \\
\hline
Matrix block dimensions \( L_x, L_y, L_z \) & 1 & ft \\
\hline
Matrix shape factor \( \sigma \) & 12 & ft\(^2\) \\
\hline
\end{tabular}
\caption{Reservoir Rock Properties}
\end{table}

The elastic rock properties were chosen so that the system was consistent with the Niobrara formation.

\begin{table}
\centering
\begin{tabular}{|l|c|l|}
\hline
 Elastic Properties & (GPa) & \\
\hline
\( K_{sm} \) & 80 & \\
\hline
\( K_{dm} \) & 23 & \\
\hline
\( K_{dfb} \) & 8,10,12 & \\
\hline
Shear modulus & 5.72 & \\
\hline
Poisson ratio & 0.25 & \\
\hline
Young modulus & 14.30 & \\
\hline
\end{tabular}
\caption{Elastic Rock Properties}
\end{table}
6.1.1.3 Fluid Rock Properties

There has been no experimental measurement available for Niobrara formations for neither relative permeabilities nor capillary pressure. Therefore, the water-oil relative permeabilities for matrix medium was modified from the experiments performed for Middle Bakken samples (Cho et al., 2016)

![Relative Permeability Curve for a Water-Oil System](image)

Figure 6-2 Relative permeability curves for water-oil system

The relative permeability for the gas-liquid system has not been performed experimentally. Thus, the end-point relative permeabilities and residual saturations were assumed taking into account the water-oil system relative permeabilities and Corey-type equations (Appendix-C). The gas-liquid relative permeabilities for matrix medium is shown in Figure 6-3.
The relative permeability for both water-oil system and gas-liquid systems for the fractures were assumed (Figure 6-4).

Similarly, capillary pressure measurements for Niobrara formation was not available. Therefore, Middle Bakken sample measurements were used for the capillary pressure measured between water and oil (Figure 6-5).
The capillary pressure for the gas-liquid system in the presence of water (Figure 6-6) was assumed using the equations (Appendix-C).

Figure 6-6 Capillary pressure curve for gas-liquid system

The capillary pressure curves for both water-oil and gas-liquid systems in the fracture medium was assumed using the equations (Figure 6-7).
6.1.2 Model Initialization

The initial condition for the flow and mechanical equilibrium such that the pressure at the fracture and matrix are same since the well was not open to production. I initialized the strain as zero; therefore, zero-displacement was generated. The details of the initial and boundary conditions are tabulated in Table 6-4:
Table 6-4 Initial Model Properties

<table>
<thead>
<tr>
<th>Reservoir Data</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Overburden thickness</td>
<td>(ft)</td>
<td>9,000</td>
</tr>
<tr>
<td>Initial bottomhole</td>
<td>(psi)</td>
<td>3729</td>
</tr>
<tr>
<td>pressure</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bubble point pressure</td>
<td>(psi)</td>
<td>3729</td>
</tr>
<tr>
<td>$S_{wf}^{initial}$</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>$S_{wm}^{initial}$</td>
<td>0.5810</td>
<td></td>
</tr>
<tr>
<td>$S_{gf}^{initial}$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$S_{gm}^{initial}$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$w_f$</td>
<td>(µm)</td>
<td>20</td>
</tr>
<tr>
<td>Roughness (C)</td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>$k_{f, eff}^{initial}$</td>
<td>(mD)</td>
<td>0.003</td>
</tr>
<tr>
<td>$\phi_f^{initial}$</td>
<td></td>
<td>0.0025</td>
</tr>
<tr>
<td>$k_m^{initial}$</td>
<td>(mD)</td>
<td>0.00000314</td>
</tr>
<tr>
<td>$\phi_m^{initial}$</td>
<td></td>
<td>0.10</td>
</tr>
</tbody>
</table>

| Production Data         |          |          |
| Bottom-hole pressure    | (psi)    | 3000     |

Kazemi et al., 1976 and Kazemi et al., 1979 conducted experiments on fracture cores and observed that fractures would not close entirely because of the presence of asperities on the fracture surface. For our modeling work, we limit the fracture closure to half of the initial width of the fracture, which is 10 µm.

6.1.3 History Match Results

Production data from eleven wells in Wishbone section for over five years were available. Before history-matching the history of a representative well from the section (Well-1N), linear flow analysis was performed to initially assume effective permeability of the SRV. The data used for the analysis and the results of the analysis were provided in Chapter 3.

The history-match was performed using a base model which was built for a single-stage hydraulic fracture and upscaled to the 50% of the stage number since around 50% of the stages are generally contribute to the flow. The simulation model was run using a pressure-controlled boundary condition and oil rate was predicted. The history-matching results displayed in Figure 6-8 and Figure 6-9 for oil rate and cumulative oil production, respectively showed a reasonable
match. The difference between the model and well history could be the assumptions in the model and the PVT model used.

Figure 6-8 History-match results for oil rate

Figure 6-9 History-match results for cumulative oil production
6.2 Sensitivity Analysis

To observe the effects of rock deformation and compressibility on production, several sensitivity analyses were conducted. First, the numerical model for 2-D, multi-phase dual-porosity model was run for a base case, system with zero bulk rock deformation, and compressible system with bulk rock deformation which included geomechanics.

![Cumulative oil production](image)

Figure 6-10 Cumulative oil production

The cumulative production (Figure 6-10) showed that the simulation runs including bulk rock deformation had the higher production than the case where there was no geomechanic effect such as compaction and porosity and permeability changes involved in the simulation runs. This production difference showed the importance of compaction during production.

As a next step, the numerical model for 2-D, fully-coupled, multi-phase, dual-porosity model was run for additional three cases to investigate the effect of bulk modulus on production. It was observed that as the bulk drained modulus of matrix blocks containing fractures, $K_{dfb}$ decreased, the compressibility of the fracture-matrix system increased. Therefore, cumulative oil production was increased (Figure 6-11).
Figure 6-11 Cumulative oil production for different $K_{\text{dff}}$ values

The final analysis was performed to investigate the effect of the non-homogeneous case where each layer has different bulk drained modulus of matrix blocks containing fractures, $K_{\text{dff}}$ (Figure 6-12). Vertical heterogeneity of the formations regarding elastic properties could be observed by the sonic log interpretation of the vertical well in the study area (Figure 6-13).

Figure 6-12 Hydraulically fractured well for a non-homogeneous case
Figure 6-13 Rock elastic properties from sonic log (RCP 2017, Tom Bratton)

Figure 6-14 displays the effect of heterogeneity in the reservoir. The non-homogeneous case had slightly more production than that of the homogeneous case.

Figure 6-14 Comparison of cumulative oil production
6.3 Discussion

Using the numerical model, two major parameters affecting oil production were identified: (1) fracture aperture reduction that is a change in rock compressibility and (2) the reduction in the formation effective permeability. Nonetheless, even though the permeability of the system decreases, the energy provided by the fracture compressibility makes up for the permeability reduction.

The numerical values for the fracture and matrix bulk moduli must satisfy $\alpha_f > \alpha_m$ because the Biot coefficient depends on the softening/hardening behavior of fractured rock total system versus the matrix rock alone. Furthermore, the Biot coefficient for the matrix-fracture system should be close to unity, and Biot coefficient for the matrix should be less than that of the fracture (Alam, 2012).
CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS

In this dissertation, fully-coupled geomechanics and multi-phase numerical simulation model was developed for unconventional shale reservoirs. Multiple continua, which included matrix and fractures, were represented by dual-porosity representation in the model. The numerical model utilized Biot’s linear poroelastic theory in transport equation and defined two distinct Biot coefficients for the matrix and the matrix-fracture system. Simulation model was constructed with the inputs obtained from the field data analysis of Niobrara formation. Following are the main conclusions of this dissertation.

- Rock deformation must be characterized by the bulk frame, which is the matrix-fracture system, and by the matrix rock without fractures. Using these parameters in a dual-porosity model is critical; otherwise, cumulative production will be underestimated.
- From simulation runs it was shown that fracture compaction (or, fracture compressibility) is a driving force during production. Thus, as the bulk modulus of the matrix-fracture system becomes smaller (indicative of the bulk-rock frame softening) production increases.
- While numerically modeling shale reservoirs, it is imperative to capture reservoir heterogeneities from petrophysics and seismic measurements to account for stress dependency of each productive layer.
REFERENCES


Archie, G., E., 1942: The Electrical Resistivity Log as an Aid in Determining some Reservoir Characteristics, Trans. AIME


APPENDIX A- FINITE DIFFERENCE DISCRETIZATION OF THE GOVERNING EQUATIONS

The 2D discretization of the pressure equation for both fracture and matrix:

- **Total Pressure Equation in the Fracture**

\[
VR_{i,k} + \frac{1}{\Delta x} \left( k_{i,j,k}^{\text{eff}} \Delta z_k + \frac{1}{\Delta z_k} \left( p_{c_{i,j,k}}^{n+1} - p_{c_{i,j,k}}^n \right) \right) = \left( k_{i,j,k}^{\text{eff}} \Delta x_i + \frac{1}{\Delta x_i} \left( p_{c_{i,j,k}}^{n+1} - p_{c_{i,j,k}}^n \right) \right) + \left( k_{i,j,k}^{\text{eff}} \Delta y_j + \frac{1}{\Delta y_j} \left( p_{c_{i,j,k}}^{n+1} - p_{c_{i,j,k}}^n \right) \right) + \left( k_{i,j,k}^{\text{eff}} \Delta z_k + \frac{1}{\Delta z_k} \left( p_{c_{i,j,k}}^{n+1} - p_{c_{i,j,k}}^n \right) \right)
\]

- **Pressure Equation in the Matrix**

\[
VR_{i,k} + \frac{1}{\Delta x} \left( k_{i,j,k}^{\text{eff}} \Delta z_k + \frac{1}{\Delta z_k} \left( p_{c_{i,j,k}}^{n+1} - p_{c_{i,j,k}}^n \right) \right) = \left( k_{i,j,k}^{\text{eff}} \Delta x_i + \frac{1}{\Delta x_i} \left( p_{c_{i,j,k}}^{n+1} - p_{c_{i,j,k}}^n \right) \right) + \left( k_{i,j,k}^{\text{eff}} \Delta y_j + \frac{1}{\Delta y_j} \left( p_{c_{i,j,k}}^{n+1} - p_{c_{i,j,k}}^n \right) \right) + \left( k_{i,j,k}^{\text{eff}} \Delta z_k + \frac{1}{\Delta z_k} \left( p_{c_{i,j,k}}^{n+1} - p_{c_{i,j,k}}^n \right) \right)
\]
- Total Pressure Equation in the Matrix

\[
\begin{align*}
\sigma_1 (k_m \lambda_{w, f/ml})_{i,k} & \left[ \left( p_{of,i,k}^{n+1} - p_{om1,i,k}^{n+1} \right) + \frac{\sigma_1}{\sigma} \gamma_w (h_{w,i,k} - h_{um1,i,k})^n \right] \\
+ \sigma_1 (k_m \lambda_{o,ml/f})_{i,k} & \left[ \left( p_{of,i,k}^{n+1} - p_{om1,i,k}^{n+1} \right) + \frac{\sigma_1}{\sigma} \gamma_o \left( h_{w,i,k} - h_{um1,i,k} \right)^n - (h_{g,i,k} - h_{gm1,i,k})^n \right] \\
+ \sigma_1 (k_m \lambda_{g, f/ml})_{i,k} & \left[ \left( p_{of,i,k}^{n+1} - p_{om1,i,k}^{n+1} \right) - \frac{\sigma_1}{\sigma} \gamma_g \left( h_{g,i,k} - h_{gm1,i,k} \right)^n + (p_{cog,f,i,k}^n - p_{cogm1,i,k}^n) \right] \\
& = \frac{1}{M_{m1}^{n+1}} \left( p_{om1,i,k}^{n+1} - p_{om1,i,k}^n \right) + \frac{\alpha_m}{\lambda + 2G} 
\end{align*}
\]

The 2D discretization of the saturation equations in the fracture:

- Water Saturation Equation in the Fracture

\[
\begin{align*}
\frac{\Delta t}{VR_{f,i,k} \phi_{f,i,k}} & \left[ \begin{array}{c}
\left( k_{f, eff} \lambda_{s, f} \right)_{i,k} + \frac{1}{\Delta z_i} \left( \frac{p_{of,i,k+1/2}^{n+1} - p_{of,i,k}^{n+1}}{\Delta z_{i+1/2}} \right) - \left( k_{f, eff} \lambda_{s, f} \right)_{i,k-1/2} + \frac{1}{\Delta x_i} \left( \frac{p_{of,i+1/2,k}^{n+1} - p_{of,i,k}^{n+1}}{\Delta x_{i+1/2}} \right)
- \left( k_{f, eff} \lambda_{s, f} \right)_{i-1/2,k} + \frac{1}{\Delta x_i} \left( \frac{p_{of,i,k-1/2}^{n+1} - p_{of,i,k}^{n+1}}{\Delta x_{i+1/2}} \right)
\end{array} \right] \\
& = S_{w_{i,k}}^{n+1} \\
\begin{array}{c}
S_{w_{i,k}}^n \left( \frac{c_{of}}{\Delta t} + \frac{c_{s, f}}{\Delta z_i} \right) \left( \frac{p_{of,i,k}^{n+1}}{\Delta t} \right) - \frac{\sigma_1}{\sigma} \gamma_w \left( h_{w,i,k} - h_{um1,i,k} \right)^n + \frac{\sigma_1}{\sigma} \gamma_o \left( h_{w,i,k} - h_{um1,i,k} \right)^n
\end{array}
\end{align*}
\]

Where,
\[
\frac{1}{M_{of}} = \frac{\phi_f}{K_{f,fl}} + \frac{(\alpha_f - \phi_f)}{K_{d,fl}} = \phi_f c_{ef} + \phi_f c_{df}
\]

\[
\frac{(\alpha_f - \phi_f)}{K_{d,fl}} = \phi_f c_{ef} \Rightarrow c_{df} = -\frac{K_{d,fl}}{\phi_f}
\]

- **Oil Saturation Equation in the Fracture**

\[
\frac{\Delta t}{VR_{i,k} \phi_{f,ijk}^n} = \sum \left[ \left( k_{f,eff} \lambda_{of} \right)_{i,k}\frac{n}{2} \left( \frac{p_{o,i,k+1}^{n+1} - p_{o,i,k}^{n+1}}{\Delta z_{i,k+1/2}} \right) \right] - \sigma_i \left( k_{w1} \lambda_{w1,of} \right)_{i,k} \left[ \frac{p_{w,i,k}^{n+1} - p_{w1,i,k}^{n+1}}{\Delta x_{i,k+1/2}} \right]
\]

- **Gas Saturation Equation in the Fracture**

\[
\frac{\Delta t}{VR_{i,k} \phi_{f,ijk}^n} = \sum \left[ \left( k_{f,eff} \lambda_{of} \right)_{i,k}\frac{n}{2} \left( \frac{p_{o,i,k}^{n+1} - p_{o,i,k}^{n+1}}{\Delta z_{i,k+1/2}} \right) \right] - \sigma_i \left( k_{w1} \lambda_{w1,of} \right)_{i,k} \left[ \frac{p_{w,i,k}^{n+1} - p_{w1,i,k}^{n+1}}{\Delta x_{i,k+1/2}} \right]
\]
The 2D discretization of the saturation equations in the matrix:

- Water Saturation Equation in the Matrix
\[ \frac{1}{M_{i,\text{el}}} = \frac{\phi_{m,i}}{K_{m,i}} (\frac{\alpha_m - \phi_m}{K_m}) = \phi_{m,i} \xi_{m,\beta} + \phi_{m,i} \xi_{m,\beta,1} \]

\[ \frac{(\alpha_m - \phi_m)}{K_m} = \phi_{m,i} \xi_{m,\beta,1} \Rightarrow c_{g,\text{el}} = \frac{K_m}{\phi_m} \]

- Oil Saturation Equation in the Matrix

\[ \frac{\Delta t}{\phi_{m,1,i,k}} \left( \sigma_1 \left( k_{m,1,i,k} \lambda_{g,m,1,f,j} \right) \right)_{i,j,k} \left[ \left( P_{o,f,1,i,k} - P_{m,1,i,k} \right) \right]_{i,j,k} + \frac{\sigma_g}{\sigma_1} \gamma_s \left( h_{g,f,1,i,k} - h_{g,m,1,i,k} \right)_{i,j,k} \left[ \left( P_{c,o,1,i,k} - P_{c,g,m,1,i,k} \right) \right]_{i,j,k} \]

- Gas Saturation Equation in the Matrix

\[ \frac{\Delta t}{\phi_{g,1,i,k}} \left( S_{g,m,1,i,k} c_{g,1,i,k} + S_{g,m,1,i,k} \right)_{i,j,k} \left[ \left( P_{o,f,1,i,k} - P_{m,1,i,k} \right) \right]_{i,j,k} \]

The 2D discretization of the rock deformation equation:

\[ w_{i,k-1/2}^{n+1} = w_{i,k+1/2}^{n} - \left( w_{i,k+1/2}^{n} - w_{i,k-1/2}^{n} \right) - \left( \alpha_m + \alpha_f \right) \frac{\Delta z_k}{\lambda + 2G} p_{o,f,1,i,k}^{n} \]

And assuming compaction is only in z direction \( \epsilon_v = \epsilon_z \):

\[ \epsilon_z^{n+1} - \epsilon_z^n = \left( \frac{\partial w}{\partial z} \right)^{n+1} = \frac{\Delta z_k \left( w_{i,k+1/2}^{n+1} - w_{i,k-1/2}^{n+1} \right)}{\Delta z_k} \]
APPENDIX B- LOGARITHMIC GRIDDING AND TIME STEPPING ALGORITHM

The Cartesian logarithmic gridding was generated using the following formula:

\[
\Delta u = \frac{1}{I_{\text{max}} - 1} \ln \left( \frac{x_i}{\Delta x_i} \right)
\]

\[
x_{i+1/2} = \Delta x_i \exp \left[ (i - 1) \Delta u \right] \quad ; \quad i = 1, 2, ..., I_{\text{max}}
\]

\[
\Delta x_i = x_{i+1/2} - x_{i-1/2} \quad ; \quad i = 2, ..., IMAX
\]

The logarithmic time stepping:

\[
\Delta \tau = \frac{1}{N} \ln \left( \frac{t}{\Delta t^0} \right)
\]

\[
t^{n+1} = \Delta t^0 \exp \left( n \Delta \tau \right) \quad ; \quad n = 0, 1, 2, ..., N
\]

Where,

\[
\Delta t^0 = \text{First time step}
\]

\[
N = \text{Total number of time steps}
\]

\[
\Delta t^n = t^{n+1} - t^n
\]
In this thesis, relative permeability data was generated using the following Corey-type equations:

\[
k_{rw}(S_w) = k_{rw}^* \left( \frac{S_w - S_{wr}}{1 - S_{wr} - S_{orw}} \right)^{nw}
\]

\[
k_{row}(S_w) = k_{row}^* \left( \frac{1 - S_w - S_{orw}}{1 - S_{wr} - S_{orw}} \right)^{now}
\]

\[
k_{rg}(S_g) = k_{rg}^* \left( \frac{S_g - S_{gc}}{1 - S_{bg} - S_{gc}} \right)^{ng}
\]

\[
k_{reg}(S_g) = k_{reg}^* \left( \frac{1 - S_g - S_{gc}}{1 - S_{bg} - S_{gc}} \right)^{nog}
\]

\[
k_{rc}(S_w, S_g) = k_{row}^* \left( \frac{k_{row}^* + k_{rw}}{k_{row}^* + k_{rg}} \right) - k_{rw} - k_{rg}
\]

(Stone-2)

\[
S_w + S_o + S_g = 1
\]

\[
S_i = S_w + S_o
\]

\[
S_i = 1 - S_g
\]

\[
p_{cwo}(S_w) = p_{th} - \alpha \ln \left( \frac{S_w - S_{wr} + 0.0001}{1 - S_{wr}} \right), \text{ First drainage}
\]

\[
p_{cwo}(S_w) = \alpha \ln \left( \frac{1 - S_{wr} - S_{orw}}{S_w - S_{wr} + 0.0001} \right), \text{ Second drainage}
\]

\[
p_{cwo}(S_w) = \alpha_1 \ln \left( \frac{1 - S_{ax} - S_{orw}}{1 - S_w - S_{orw} + 0.0001} \right), \text{ First imbibition for } S_{ax} < S_w < 1 - S_{orw}
\]

\[
p_{cwo}(S_w) = \alpha_2 \ln \left( \frac{1 - S_{ax} - S_{wr}}{S_w - S_{wr} + 0.0001} \right), \text{ First imbibition for } S_{wr} < S_w < 1 - S_{ax}
\]

\[
\alpha_2 = \frac{S_{wx} - S_{ar}}{1 - S_{wx} - S_{orw}} \alpha_1
\]

\[
S_{wx} = 1 - S_{ax}
\]

\[
p_{cog}(S_g) = p_{th} - \alpha \ln \left( \frac{S_i - S_{bg} + 0.0001}{1 - S_{bg}} \right)
\]