UNCONVENTIONAL RESERVOIR ENGINEERING PROJECT
COLORADO SCHOOL OF MINES
UREP CSM

Research Summary

A New Superposition Time to Account for Pressure-Dependent Rock and Fluid Properties in Tight-Gas Reservoirs

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Motivation

Most conventional models used in the analysis of tight-gas well data assume that pseudopressure transformation linearizes the problem

This approach is suitable if the gas viscosity-compressibility product remains approximately constant throughout the production history to be analyzed

There is enough evidence that most tight-gas production data display effects of pressure-dependence of the viscosity-compressibility product

Problem

Gas Diffusion Equation in terms of Pressure

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{p}{\mu Z}\frac{\partial p}{\partial r}\right)=\frac{\phi c_t}{2.637\times10^{-4}k}\frac{p}{Z}\frac{\partial p}{\partial t}$$

Pseudopressure

$$m(p)=2\int_{p_0}^{p}\frac{p'}{\mu Z}dp'$$

Gas Diffusion Equation in terms of Pseudopressure

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial m}{\partial r}\right)=\frac{\phi c_t \mu}{2.637\times10^{-4}k}\frac{\partial m}{\partial t}$$

functions of pressure

Approach

Assume 1D (linear) Flow and Consider

$$\frac{\partial^2 \Delta m}{\partial y^2}=(1+\omega)\frac{1}{\eta_i}\frac{\partial \Delta m}{\partial t}$$

where

$$\omega=\omega(y,t)=\frac{\eta_i-\eta}{\eta}=\frac{(\phi\mu c)_i-(\phi\mu c)}{(\phi\mu c)}$$

and

$$\eta=\frac{2.637\times10^{-4}k}{\phi\mu c}$$

Problem

Diffusion Equation

$$\frac{\partial^2 \Delta m}{\partial y^2} = (1+\omega)\frac{1}{\eta_i}\frac{\partial \Delta m}{\partial t}$$

Initial and Boundary Conditions

$$\Delta m(y, t \to 0) = 0$$

$$\Delta m(y \to \infty, t) = 0$$

and

$$\frac{\partial \Delta m}{\partial y}(y=0,t) = -\frac{2844\pi qT}{2khx_f}$$

Perturbation Problem

Diffusion Equation in Perturbation Form

$$\frac{\partial^2 \Delta m}{\partial y^2} = (1+\varepsilon\omega)\frac{1}{\eta_i}\frac{\partial \Delta m}{\partial t} \qquad \varepsilon = \begin{cases} 0 & \text{Linear} \\ 1 & \text{Non-Linear} \end{cases}$$

Solution can be assumed in the following form

$$\Delta m = \Delta m^0 + \sum_{k=1}^{\infty} \varepsilon^k \Delta m^k$$

Substituting

$$\left(\frac{\partial^2 \Delta m^0}{\partial y^2} - \frac{1}{\eta_i}\frac{\partial \Delta m^0}{\partial t}\right) + \varepsilon\left(\frac{\partial^2 \Delta m^1}{\partial y^2} - \frac{1}{\eta_i}\frac{\partial \Delta m^1}{\partial t} - \frac{\omega^0}{\eta_i}\frac{\partial \Delta m^0}{\partial t}\right)$$

$$+\varepsilon^2\left(\frac{\partial^2 \Delta m^2}{\partial y^2} - \frac{1}{\eta_i}\frac{\partial \Delta m^2}{\partial t} - \frac{\omega^1}{\eta_i}\frac{\partial \Delta m^1}{\partial t}\right) + \cdots = 0$$

Perturbation Problem

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$$+\varepsilon^2\left(\frac{\partial^2 \Delta m^2}{\partial y^2} - \frac{1}{\eta_i}\frac{\partial \Delta m^2}{\partial t} - \frac{\omega^1}{\eta_i}\frac{\partial \Delta m^1}{\partial t}\right) + \cdots = 0$$

Perturbation Problem

We have

$$\left(\frac{\partial^2 \Delta m^0}{\partial y^2} - \frac{1}{\eta_i}\frac{\partial \Delta m^0}{\partial t}\right) + \varepsilon\left(\frac{\partial^2 \Delta m^1}{\partial y^2} - \frac{1}{\eta_i}\frac{\partial \Delta m^1}{\partial t} - \frac{\omega^0}{\eta_i}\frac{\partial \Delta m^0}{\partial t}\right)$$

$$+\varepsilon^2\left(\frac{\partial^2 \Delta m^2}{\partial y^2} - \frac{1}{\eta_i}\frac{\partial \Delta m^2}{\partial t} - \frac{\omega^1}{\eta_i}\frac{\partial \Delta m^1}{\partial t}\right) + \cdots = 0$$

$\Delta m^0,\ \Delta m^1,\ \Delta m^2,\ \cdots$ are the solutions of

$$\frac{\partial^2 \Delta m^0}{\partial y^2} - \frac{1}{\eta_i}\frac{\partial \Delta m^0}{\partial t} = 0$$

$$\frac{\partial^2 \Delta m^1}{\partial y^2} - \frac{1}{\eta_i}\frac{\partial \Delta m^1}{\partial t} - \frac{\omega^0}{\eta_i}\frac{\partial \Delta m^0}{\partial t} = 0$$

$$\frac{\partial^2 \Delta m^2}{\partial y^2} - \frac{1}{\eta_i}\frac{\partial \Delta m^2}{\partial t} - \frac{\omega^1}{\eta_i}\frac{\partial \Delta m^1}{\partial t} = 0$$

$$\vdots$$

$$\frac{\partial^2 \Delta m^k}{\partial y^2} - \frac{1}{\eta_i}\frac{\partial \Delta m^k}{\partial t} - \frac{\omega^{k-1}}{\eta_i}\frac{\partial \Delta m^{k-1}}{\partial t} = 0$$

Perturbation Solution

Green's Function Solutions

$$\Delta m^0(y,t) = \frac{2844T\pi}{k}\eta_i \int_0^t \tilde{q}(t')S(y,t-t')dt'$$

$$\Delta m^1(y,t) = \frac{2844T\pi}{k}\eta_i \int_0^t \tilde{q}(t')\kappa^1(t')S(y,t-t')dt' - 2\eta_i\left[\frac{\partial\omega^0}{\partial\Delta m^0}\left(\frac{\partial\Delta m^0}{\partial y}\right)^2\right]_{y,t}$$

$$\Delta m^2(y,t) = \frac{2844T\pi}{k}\eta_i \int_0^t \tilde{q}(t')\kappa^2(t')S(y,t-t')dt'$$

$$-2\eta_i\left\{\left[\frac{\partial\omega^1}{\partial\Delta m^1}\left(\frac{\partial\Delta m^1}{\partial y}\right)^2\right]_{y,t} - \left[\omega^1\frac{\partial\omega^0}{\partial\Delta m^0}\left(\frac{\partial\Delta m^0}{\partial y}\right)^2\right]_{y,t}\right\}$$

$$\Delta m^k(y,t) = -2\eta_i\left\{\left[\frac{\partial\omega^{k-1}}{\partial\Delta m^{k-1}}\left(\frac{\partial\Delta m^{k-1}}{\partial y}\right)^2\right]_{y,t} - \left[\omega^{k-1}\frac{\partial\omega^{k-2}}{\partial\Delta m^{k-2}}\left(\frac{\partial\Delta m^{k-2}}{\partial y}\right)^2\right]_{y,t}\right\} \quad k \geq 3$$

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Perturbation Solution

Green's Function Solutions

$$\Delta m(y,t) = \Delta m^0 + \Delta m^1 + \Delta m^2 + \sum_{k=3}^{\infty}\Delta m^k$$

$$= \frac{2844T\pi}{k}\eta_i \int_0^t \tilde{q}(t')\left[1+\kappa(t')\right]S(y,t-t')dt' - 2\eta_i\left[\frac{\partial\omega^1}{\partial\Delta m^1}\left(\frac{\partial\Delta m^1}{\partial y}\right)^2\right]_{y,t}$$

$$-2\eta_i\sum_{k=2}^{\infty}\left\{\left[\frac{\partial\omega^{k-1}}{\partial\Delta m^{k-1}}\left(\frac{\partial\Delta m^{k-1}}{\partial y}\right)^2\right]_{y,t} - \left[\omega^{k-1}\frac{\partial\omega^{k-2}}{\partial\Delta m^{k-2}}\left(\frac{\partial\Delta m^{k-2}}{\partial y}\right)^2\right]_{y,t}\right\}$$

$$S = S(y,t-t') = \frac{1}{2\sqrt{\pi\eta_i(t-t')}}\exp\left[-\frac{y^2}{4\eta_i(t-t')}\right]$$

$$\kappa(t) = (1-\omega^1)_{y=0}\,\kappa^1(t) \qquad \kappa^1(t') = -\left[\omega^0 + \left(1-\frac{\overline{\Delta m^0}}{\Delta m^0}\right)\frac{\partial\omega^0}{\partial\ln\Delta m^0}\right]$$

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Perturbation Solution

On the fracture plane

$$\Delta m(0,t) = \frac{2844T\sqrt{\pi\eta_i}}{2k}\int_0^t \tilde{q}(t')\left[1+\kappa(t')\right]\frac{dt'}{\sqrt{(t-t')}}$$
$$+2\eta_i\left(\frac{2844T\pi}{k}\right)^2\left(\frac{\omega^1}{\Delta m^0}\frac{\partial\omega^0}{\partial\ln\Delta m^0}\right)_{0,t}\left[\tilde{q}(t)\right]^2$$

Discretizing and arranging

$$\frac{\Delta m(0,t)}{q(t)} = \frac{2844T\sqrt{\pi\eta_i}}{2x_f kh}\left\{\frac{q_1}{q(t)}(1+\kappa_1)\sqrt{t} + \sum_{i=1}^{n}\left[q_{i+1}(1+\kappa_{i+1}) - q_i(1+\kappa_i)\right]\frac{\sqrt{t-t_i}}{q(t)}\right\}$$
$$+\frac{\eta_i}{2}\left(\frac{2844T\pi}{x_f kh}\right)^2\left(\frac{\omega^1}{\Delta m^0}\frac{\partial\omega^0}{\partial\ln\Delta m^0}\right)_{0,t} q(t)$$

Superposition Time

Consider

$$\frac{\Delta m(0,t)}{q(t)} = \frac{2844T\sqrt{\pi\eta_i}}{2x_f kh}\left\{\frac{q_1}{q(t)}(1+\kappa_1)\sqrt{t} + \sum_{i=1}^{n}\left[q_{i+1}(1+\kappa_{i+1}) - q_i(1+\kappa_i)\right]\frac{\sqrt{t-t_i}}{q(t)}\right\}$$
$$+\frac{\eta_i}{2}\left(\frac{2844T\pi}{x_f kh}\right)^2\left(\frac{\omega^1}{\Delta m^0}\frac{\partial\omega^0}{\partial\ln\Delta m^0}\right)_{0,t} q(t)$$

and define

$$t_{\sup} = \frac{q_1}{q(t)}(1+\kappa_1)\sqrt{t} + \sum_{i=1}^{n}\left[q_{i+1}(1+\kappa_{i+1}) - q_i(1+\kappa_i)\right]\frac{\sqrt{t-t_i}}{q(t)}$$

Then

$$\frac{\Delta m(0,t)}{q(t)} = \frac{2844T\sqrt{\pi\eta_i}}{2x_f kh}t_{\sup} + \frac{\eta_i}{2}\left(\frac{2844T\pi}{x_f kh}\right)^2\left(\frac{\omega^1}{\Delta m^0}\frac{\partial\omega^0}{\partial\ln\Delta m^0}\right)_{0,t} q(t)$$

Conclusions

- It is not possible to define a simple superposition time which yields a linear equation for the normalized pseudopressure as a function of superposition time
- If the dependency of viscosity-compressibility product on pressure is weak, then the additional non-linear term may be negligible and a linear relation may be obtained
- The new solution may be used to develop a model-based computerized (regression) analysis method