UNCONVENTIONAL RESERVOIR ENGINEERING PROJECT
COLORADO SCHOOL OF MINES

Progress Report

Fractional Diffusion in Naturally Fractured Unconventional Reservoirs

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Non-Local Anomalous Diffusion

Diffusion is the result of the random Brownian motion of individual particles.

The mean square displacement of a particle is a linear function of time

$$\sigma_r^2 \sim Dt$$

For the Brownian motion, the probability density function in space, evolving in time, is of the Gaussian type

Non-Local Anomalous Diffusion

However, a convincing number of works have indicated anomalous diffusion in which the mean square variance grows faster (superdiffusion) or slower (subdiffusion) than that in a Gaussian diffusion process.

Thus, a general relationship between the mean square variance and time is given by

$$\sigma_r^2 \sim Dt^\alpha$$

$\alpha = 1$ Normal Diffusion
$\alpha \neq 1$ Anomalous Diffusion
$\alpha > 1$ Superdiffusion
$\alpha < 1$ Subdiffusion.

Non-Local Anomalous Diffusion

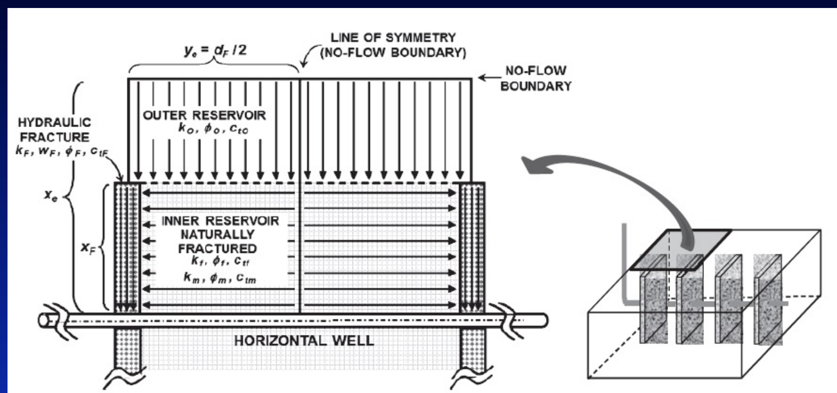
Flux Equation for normal diffusion: $v = -D \dfrac{\partial p}{\partial x}$

Flux Equation for non-local anomalous diffusion (time fractional): $v = -D_f \dfrac{\partial^{1-\alpha}}{\partial t^{1-\alpha}} \dfrac{\partial p}{\partial x}$

Flux Equation for non-local anomalous diffusion (time and space fractional): $v = D_f \partial_t^{1-\gamma} \left(\dfrac{\partial^\beta p}{\partial x^\beta} \right), \quad 0 < \gamma, \quad \beta < 1$

Model Description

Trilinear Flow Model



(Brown, M., 2009)

Outer Reservoir Solution

- Flow from the outer reservoir to the inner reservoir is assumed to be linear

$$k_O \frac{\partial^2 \Delta p_O}{\partial x^2} = \phi_O c_{tO} \mu \frac{\partial p_O}{\partial t} \qquad \frac{\partial^2 p_{OD}}{dx_D^2} - \frac{s}{\eta_{OD}} p_{\ldots}$$

Outer Boundary Condition : $\left(\frac{dp_{OD}}{dx_D}\right)_{x_D = x_{eD}} = 0$

Inner Boundary Condition : $(p_{OD})_{x_D=1} = (p_{ID})_{x_D=1}$

Solution for Outer Res. : $(p_{OD})_{x_D=1} = (p_{ID})_{x_D=1} \dfrac{\cosh\left(\sqrt{s/\eta_{OD}}\,(x_{eD} - x_D)\right)}{\cosh\left(\sqrt{s/\eta_{OD}}\,(x_{eD} - 1)\right)}$

Inner Reservoir Solution

- Non-Local Flux

1. Only Time Fractional (Bounded)
2. Space and Time Fractional (Infinite)

Inner Reservoir Solution (Time Fractional)

Flux Term

$$v = -\lambda_\alpha \frac{\partial^{1-\alpha}}{\partial t^{1-\alpha}} \left(\frac{\partial \Delta p_I}{\partial x} \right)$$

Diffusion Eq

$$\frac{\partial}{\partial x}\left\{ \lambda_\alpha \frac{\partial \Delta p_I}{\partial x} \right\} + \frac{\partial}{\partial y}\left\{ \lambda_\alpha \frac{\partial \Delta p_I}{\partial y} \right\} = (\phi c_t)_I \frac{\partial^\alpha}{\partial t^\alpha} \Delta p_I$$

Outer Boundary Condition :

$$\left(\frac{dp_{ID}}{dy_D} \right)_{y_D = y_{eD}} = 0$$

Inner Boundary Condition :

$$(p_{ID})_{y_D = w_D/2} = (p_{FD})_{y_D =}$$

Solution for Inner Res. :

$$(p_{ID})_{y_D = w_D/2} = (p_{FD})_{y_D = w_D/2} \cosh[\sqrt{a_0}\,(y_{eD} - y_D)] / \cosh[\sqrt{a_0}\,(y_{eD} - w_D/2)]$$

Hydraulic Fracture and Wellbore Solution

$$\frac{\partial^2 \Delta p_F}{\partial x^2} + \frac{\partial^2 \Delta p_F}{\partial y^2} = \frac{(\phi c_t)_F \mu}{k_F} \frac{\partial \Delta P_F}{\ldots}$$

$$\frac{d^2 p_{FD}}{dx_D^2} + \frac{2}{w_D}\frac{\lambda_I}{\lambda_F}\left(\frac{\eta_I}{x_F^2}\right)^{1-\alpha} s^{1-\alpha}\left(\frac{dp_{ID}}{dy_D}\right)_{y_D = w_D/2} = \frac{1}{\eta_{FD}} s p_{FD}$$

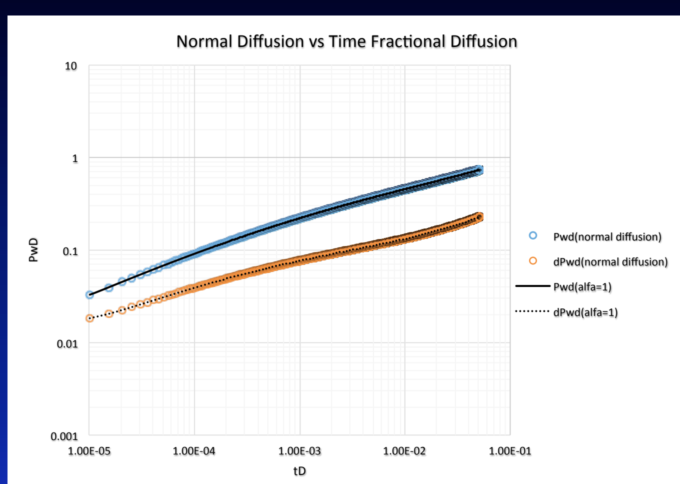
Outer Boundary Condition : $\left(\frac{dp_{FD}}{dx_D}\right)_{x_D=1} = 0$

Inner Boundary Condition : $\left(\frac{dp_{FD}}{dx_D}\right)_{x_D=0} = -\frac{\pi}{sC_{FD}}$

Solution for Hyd. Fracture : $p_{FD} = \frac{\pi}{sC_{FD}\sqrt{\alpha_F}} \frac{\cosh[\sqrt{\alpha_F}(1-x_D)]}{\sinh(\sqrt{\alpha_F})}$

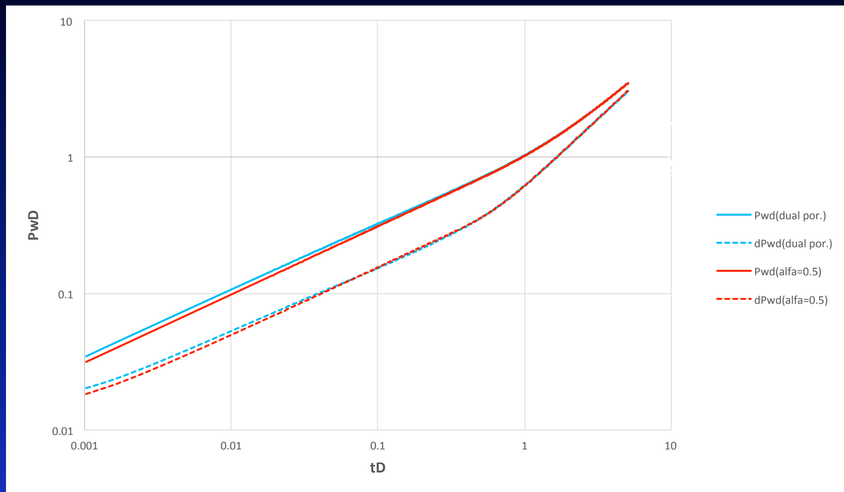
Wellbore Pressure Solution : $p_{wD} = (p_{FD})_{x_D=0} = \frac{\pi}{sC_{FD}\sqrt{\alpha_F}} \tanh(\sqrt{\alpha_F})$

Results

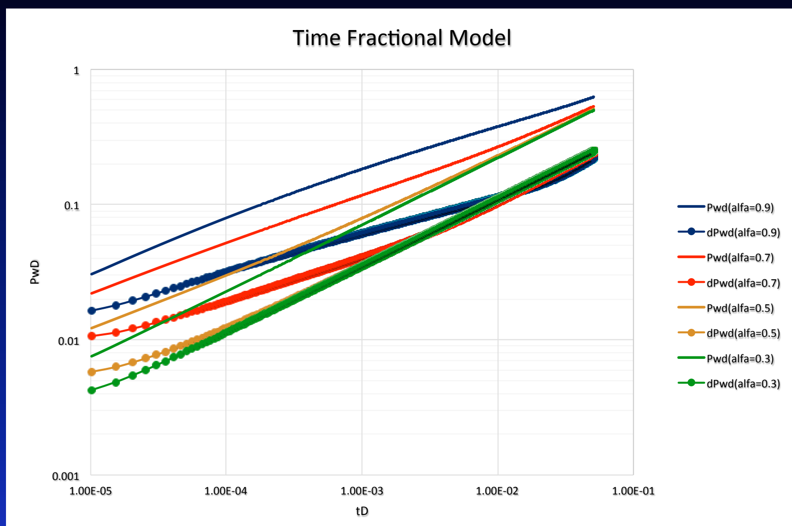


Results

Dual Porosity vs Time Fractional Model



Comparison For Different "α" Values



Inner Reservoir Solution (Space & Time Fractional)

Flux Term:

$$v = -\lambda_{\alpha,\beta} \frac{\partial^{1-\alpha}}{\partial t^{1-\alpha}} \left(\frac{\partial^{\beta}}{\partial x^{\beta}} \Delta p_I \right)$$

Diffusion Equation:

$$\frac{\partial}{\partial x} \left\{ \lambda_{\alpha,\beta} \frac{\partial^{\beta}}{\partial x^{\beta}} \Delta p_I \right\} + \frac{\partial}{\partial y} \left\{ \lambda_{\alpha,\beta} \frac{\partial^{\beta}}{\partial y^{\beta}} \Delta p_I \right\} = (\phi c_t)_I \frac{\partial^{\alpha}}{\partial t^{\alpha}} \Delta p_I$$

Continuity of flux at the boundary of the inner and outer res:

$$\lambda_{\alpha,\beta} \frac{\partial^{1-\alpha}}{\partial t^{1-\alpha}} \left(\frac{\partial^{\beta}}{\partial x^{\beta}} \Delta p_I \right)\Big|_{x=x_F} = \frac{k_o}{\mu} \left(\frac{\partial}{\partial x} \Delta p_o \right)\Big|_{x=x_F}$$

Solution for Inner Res. :

$$p_{ID}(y_D,s) = p_{ID}(0,s) E_{\beta+1}\left(\alpha_O y_D^{\beta+1}\right) + \left[\frac{\partial^{\beta}}{\partial y_D^{\beta}} p_{ID}(y_D,s) \right]_{y_D=0} y_D^{\beta} E_{\beta+1,\beta+1}\left(\alpha_O y_D^{\beta+1}\right)$$

Comments and Questions

- An alternative to dual porosity models
- Needs physical interpretation
- What does the permeability in fractional diffusion model refer to?
- What is the meaning of α?