



**UNCONVENTIONAL RESERVOIR ENGINEERING PROJECT**  
**COLORADO SCHOOL OF MINES**



## Research Report

# A Spectral Solution for the Analysis of Multifractured Tight-Gas Wells with Large Viscosity-Compressibility Variation and Pressure-Dependent Permeability

Caglar Komurcu, Colorado School of Mines  
Leslie Thompson, Cimarex Energy



**UNCONVENTIONAL RESERVOIR ENGINEERING PROJECT**

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# Problem Statement

## Analysis of multifractured tight-gas wells

There are two approaches used in the analysis of multifractured tight-gas well performances:

- Numerical simulation
- Analytical modeling

There are known problems with both approaches

The shortcomings of the analysis approaches make the results questionable for most tight-gas well cases



# Problem Statement

## Numerical Simulation

Current commercial simulators are not able to capture all of the physics, and even if they did, their accuracy would be questionable.

Finite-difference methods are inherently approximate; they replace partial differential equations by local algebraic difference equations

To obtain accurate solutions, very fine grids and very small time steps need to be used

Available data usually do not justify the use of detailed numerical simulators



# Problem Statement

## Analytical Modeling

Continuity equation for flow of real gases in porous media is non-linear

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{p}{\mu Z} \frac{\partial p}{\partial r} \right) = \frac{\phi}{2.637 \times 10^{-4} k} \frac{\partial(p/Z)}{\partial t}$$

For analytical modeling, solutions of the continuity equation must be obtained

This requires linearization of the continuity equation



# Problem Statement

## Analytical Modeling

A common approach to linearize the continuity equation for real-gas flow is to define a real gas pseudopressure:

$$m(p) = 2 \int_{p_o}^p \frac{p'}{\mu Z} dp' \quad \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial m}{\partial r} \right) = \frac{\phi c_{i, \mu}}{2.637 \times 10^{-4} k} \frac{\partial m}{\partial t}$$

Still nonlinear because compressibility and viscosity are functions of pressure

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial m}{\partial r} \right) = \frac{\phi c_{i, \mu_i}}{2.637 \times 10^{-4} k} \frac{\partial m}{\partial t}$$



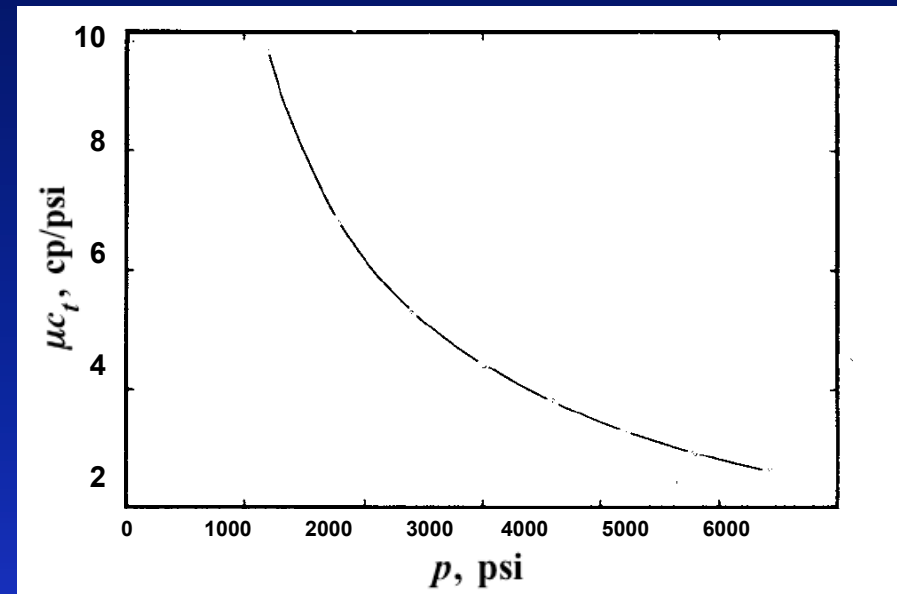
# Problem Statement

## Analytical Modeling

Pseudopressure linearization assumes that the change in viscosity-compressibility product is negligible

This assumption is reasonable at

- relatively high pressures
- small pressure changes
- infinite-acting behavior
- pressure-transient analysis
- ...
- ...



# Problem Statement

## Analytical Modeling

The assumptions of pseudopressure linearization may be met in the analyses of some conventional reservoirs

In tight-gas reservoirs pressure drops are very large; so we have large changes in compressibility and viscosity.

Additional complexities may exist due to

- Pressure dependent permeability
- Non-Darcy flow effects
- Multiphase flow effects
- The Klinkenberg effect



# Objectives

## Objectives of the Research

Document the problems encountered and errors incurred in the analysis of tight-gas well performances by pseudopressure linearization

Present an analytical and a semi-analytical spectral solution to analyze the performances of fractured wells in tight-gas reservoirs

Demonstrate the analysis with the new solution and highlight the improvement in results





# Scope of Research

## Scope of the Research

Phase I – Investigation of the variable compressibility and viscosity product effects on data analysis.

$$C_1 \frac{\partial^2 m(p)}{\partial x^2} = \phi c_g \mu_g \frac{\partial m(p)}{\partial t}$$

Phase II – Development of a spectral solution of the gas diffusivity equation and incorporation of the essential physics of flow in tight-gas reservoirs

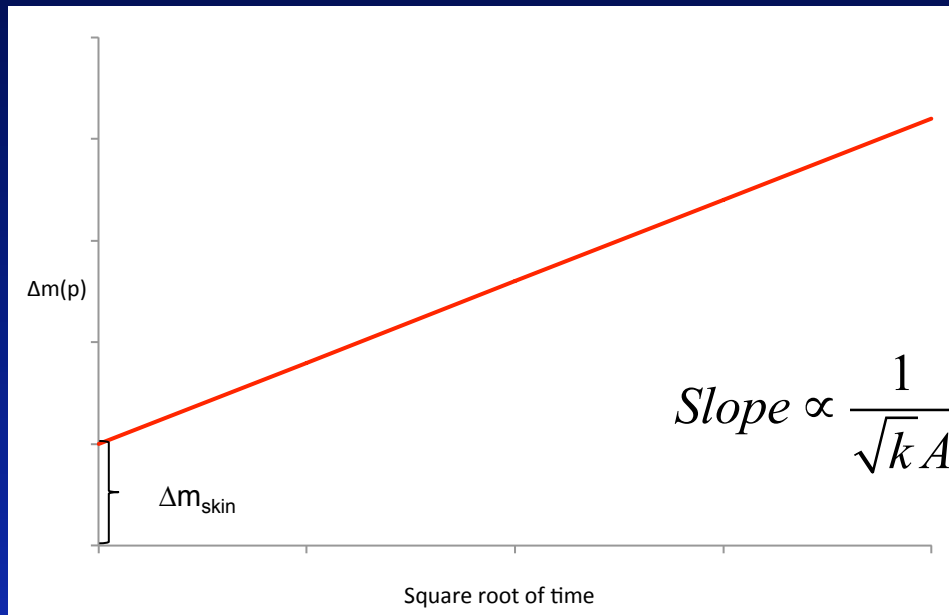
Phase III – Demonstration of the data analysis with the new solution



# Status

## Phase I

### Constant Rate Solution



- Plot of pseudopressure drop versus square root of time is a straight line
- Slope gives matrix properties
- Intercept gives skin



# Status

## Phase I

### Tight-Gas Production Data Analysis

- Tight-gas wells produce at variable rate-variable pressure
- After the initial flow period, pressure may stabilize while the rate declines

If the pseudopressure approach linearizes the real-gas continuity equation, then convolution (Duhamel's equation/superposition) can be applied to develop solutions and analysis procedures



# Status

## Phase I

### Tight-Gas Production Data Analysis

There are two common methods of applying the superposition solution

- Rate normalization
- Superposition time



# Status

## Phase I

### Tight-Gas Production Data Analysis

#### *Rate normalization*

Pressure response at any time is mostly due to the most recent behavior;

If wells produce at pressures that vary slowly with time

$$\frac{\Delta m_w(t)}{q(t)} \approx \frac{\Delta m_{wc}}{q_c(t)}$$

will follow the constant pressure production solution.

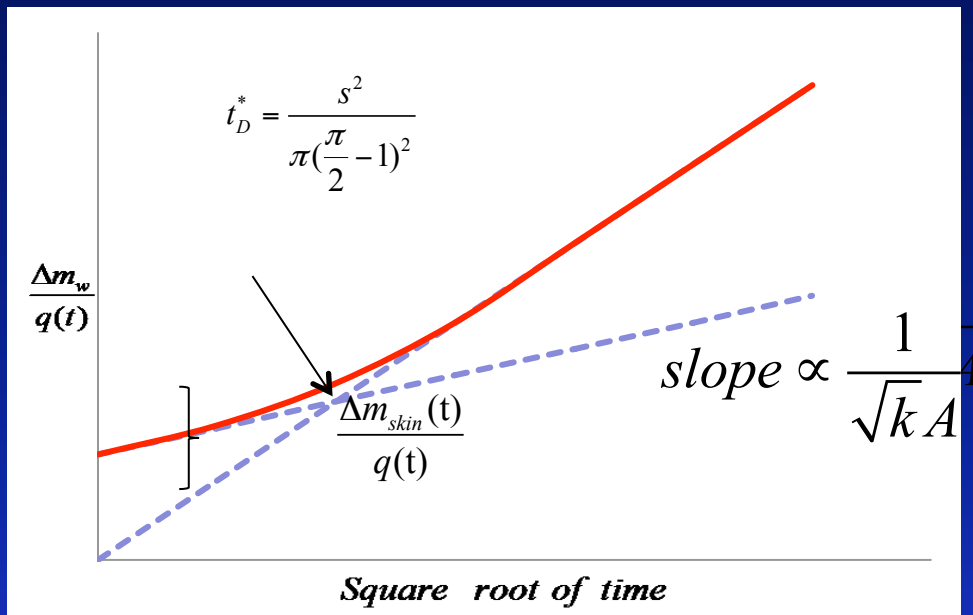


# Status

## Phase I

### Tight-Gas Production Data Analysis

#### Rate-Normalization & Constant Pressure Solution (Wattenbarger)



• Solution is not a straight line, but asymptotic to straight lines at “early (<90 days)” and “late” times

- Early time  $\frac{1}{[q_{Dc}]_{ET}} \approx \sqrt{\pi t_D} + s$

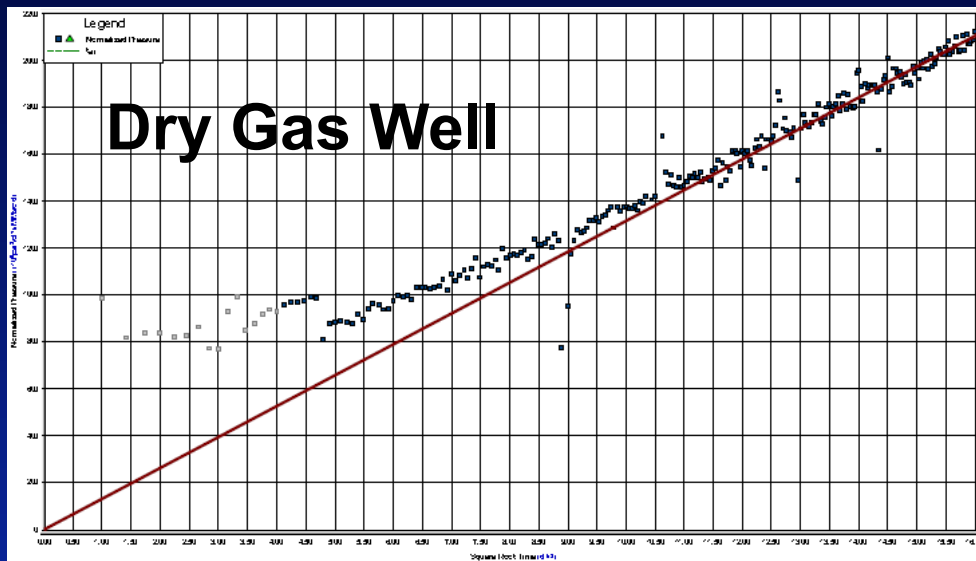
- Late time  $[q_{Dc}]_{LT} \approx \frac{2}{\pi \sqrt{\pi t_D}}$



# Status

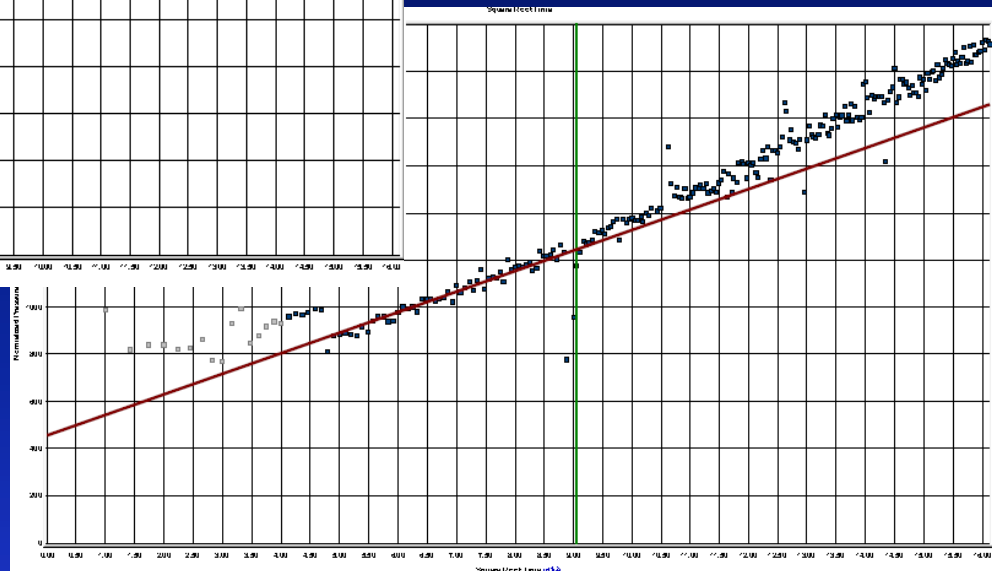
## Phase I

### Tight-Gas Production Data Analysis



Rate-Normalization & Constant Pressure Solution (Wattenbarger)

Ratio of late to early time slopes is 1.57



# Status

## Phase I

### Tight-Gas Production Data Analysis

*Superposition time & Constant rate solution:*

$$m_{Dc}(t_D) = \sqrt{\pi t_D} + s$$

Apply to the superposition solution:

$$m_{Dc}(t_D) = \int_0^{t_D} q_D(\tau) m'_{Dc}(t_D - \tau) d\tau$$

Superposition time:

$$t_s = \frac{q(t_1)\sqrt{t_n}}{q(t_n)} + \sum_{k=2}^n \frac{q(t_k) - q(t_{k-1})}{q(t_n)} \sqrt{t_n - t_{k-1}}$$

Normalized pseudopressure vs. superposition time should follow the constant rate solution

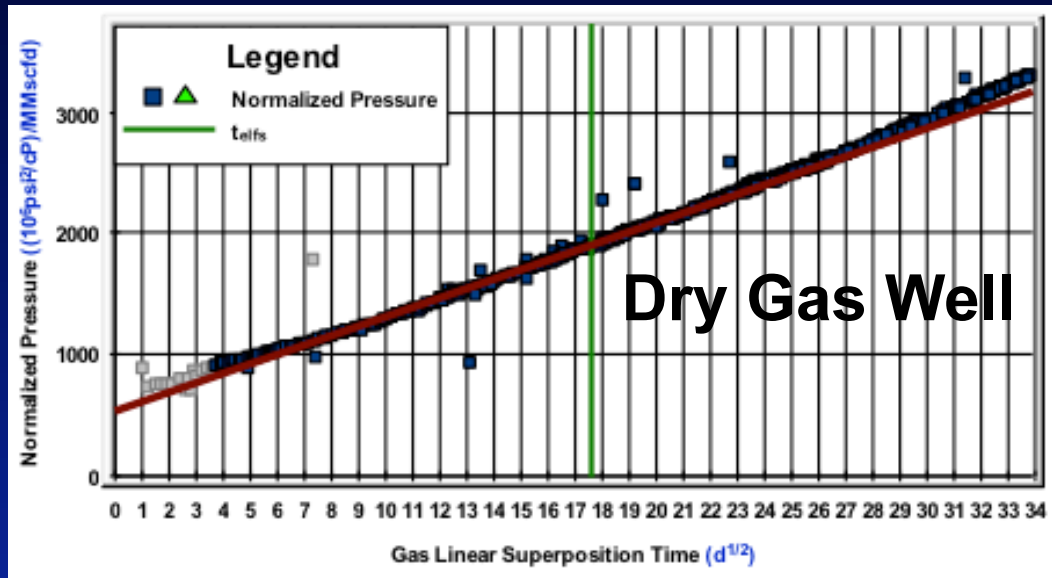




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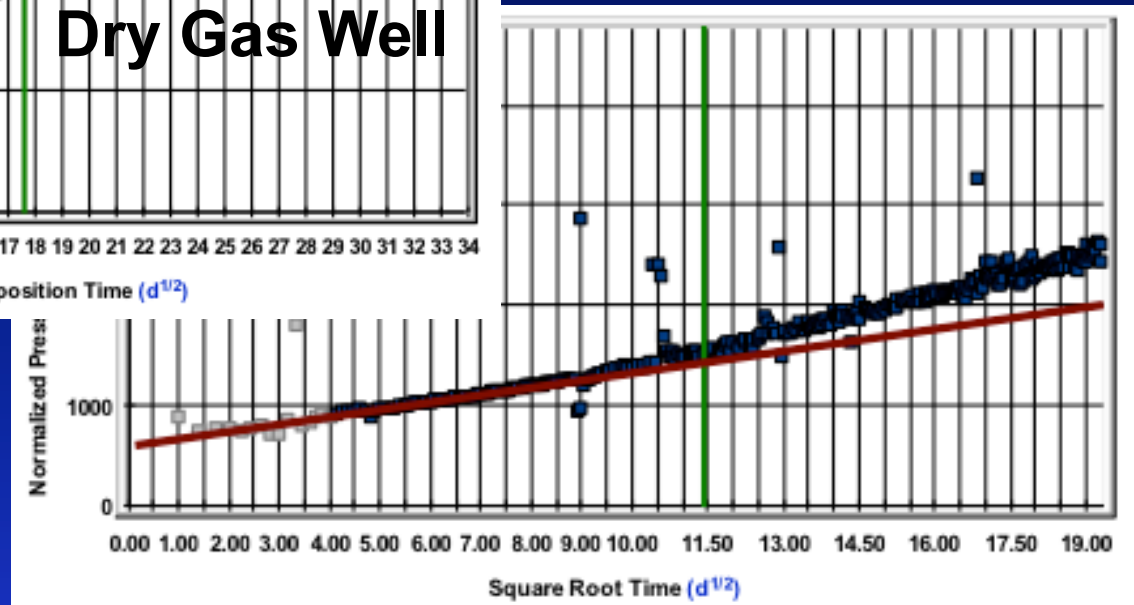
## Phase I

### Tight-Gas Production Data Analysis



*Superposition time & Constant rate solution*

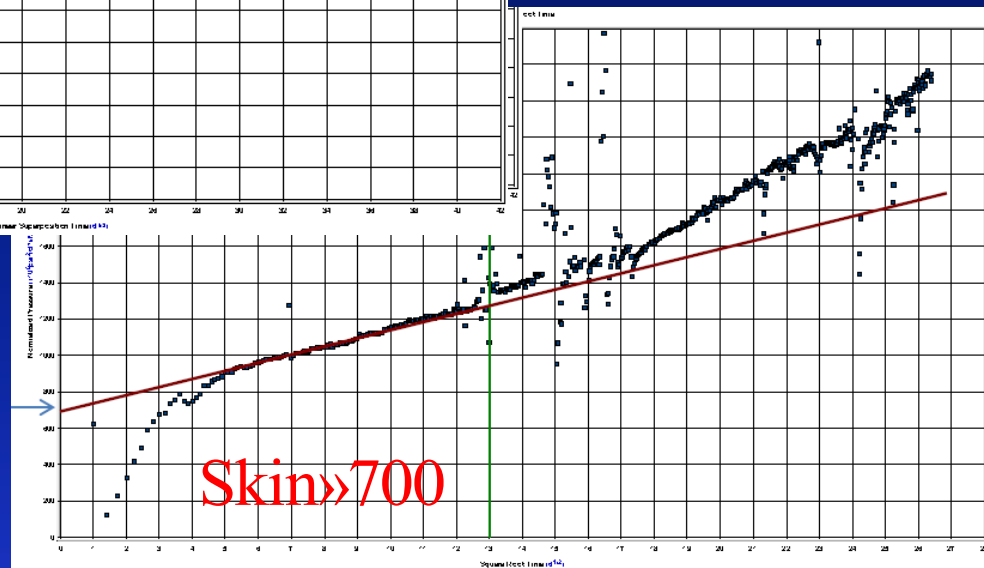
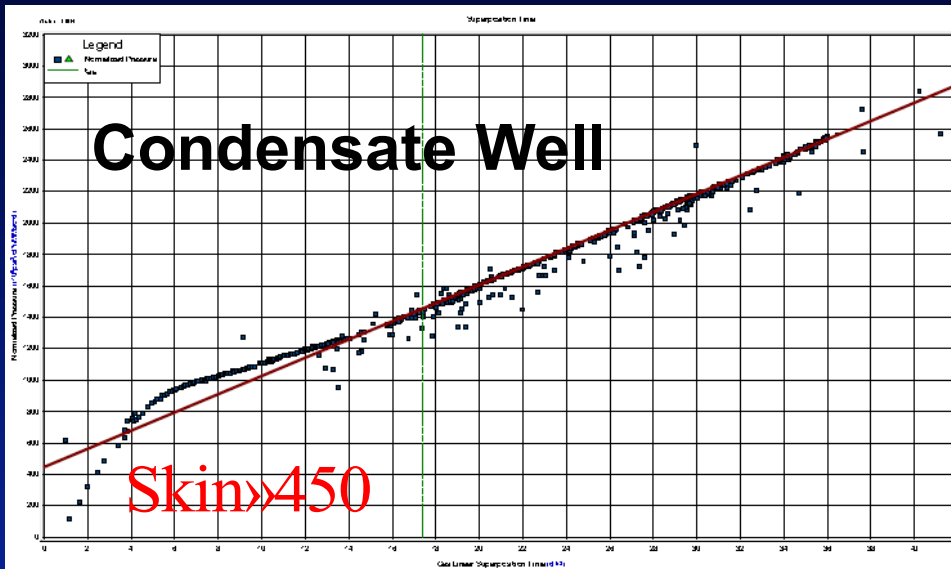
Early square root of time and superposition time and slopes should be the same



# Status

## Phase I

### Tight-Gas Production Data Analysis



# Status

## Phase II

### Spectral Methods

Assumption: The solution of nonlinear diffusion equations is given in the form of an infinite series

Solution: Substitute the infinite series into the diffusion equation and solve for the coefficients

Approach: Approximate the infinite series solution by a truncated Chebyshev series and solve for the (time dependent) coefficients

$$p_M(x, t) \approx \sum_{k=0}^N c_k p(t) \cos(k \arccos x)$$

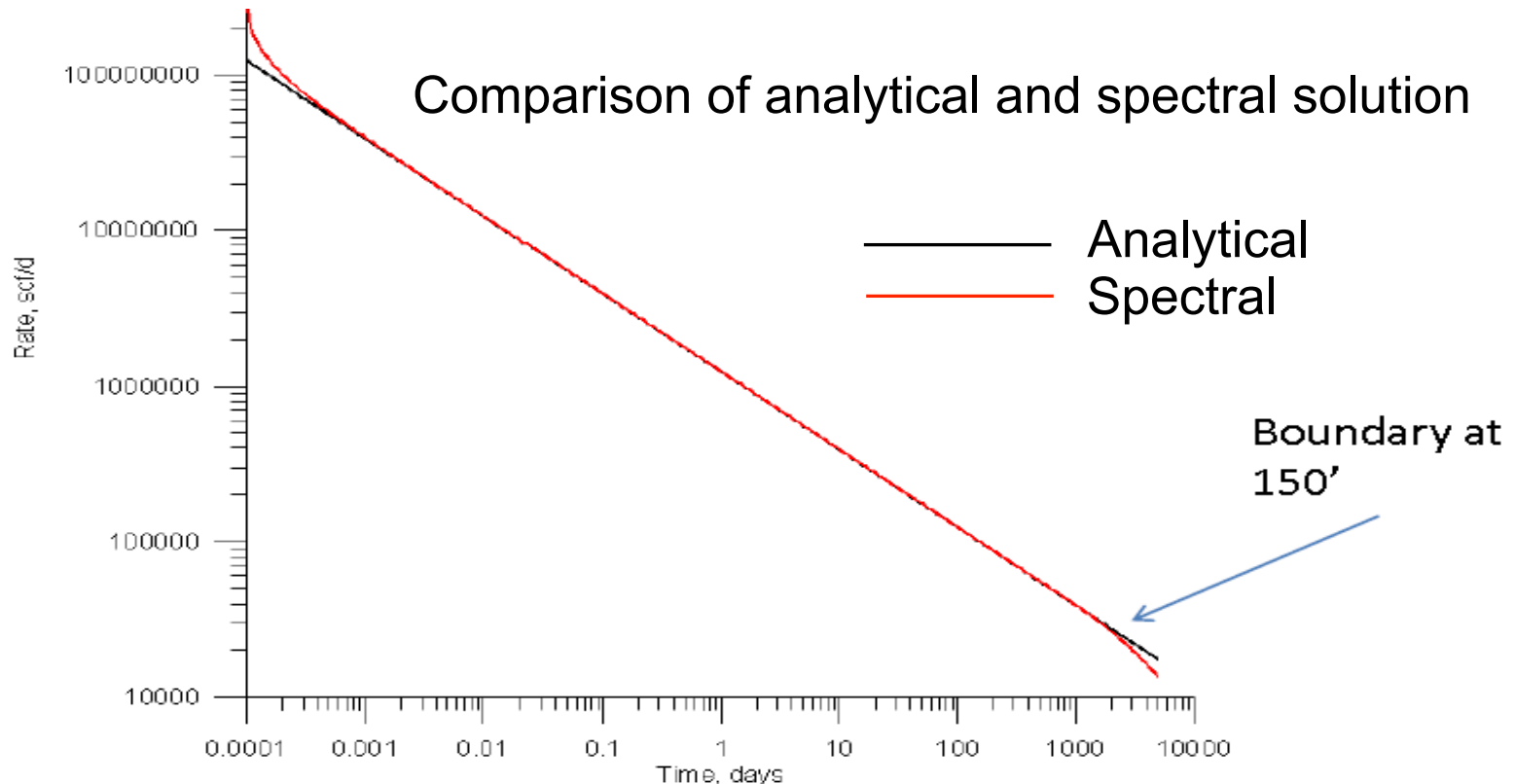


# Status

## Phase II

### Variable compressibility effect on constant pressure results

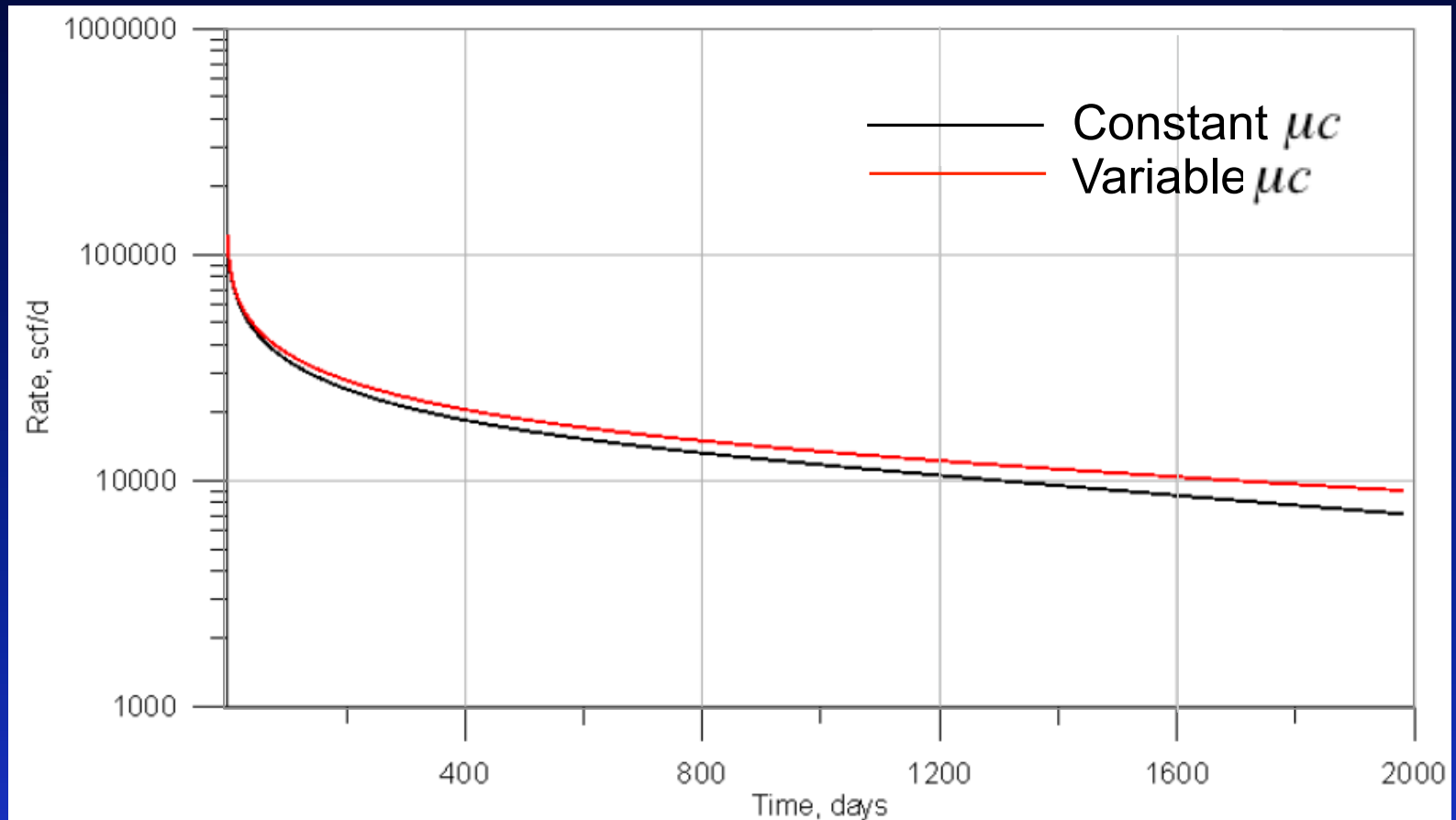
We have an analytical solution and a spectral solution for constant pressure production incorporating variable compressibility effect



# Status

## Phase II

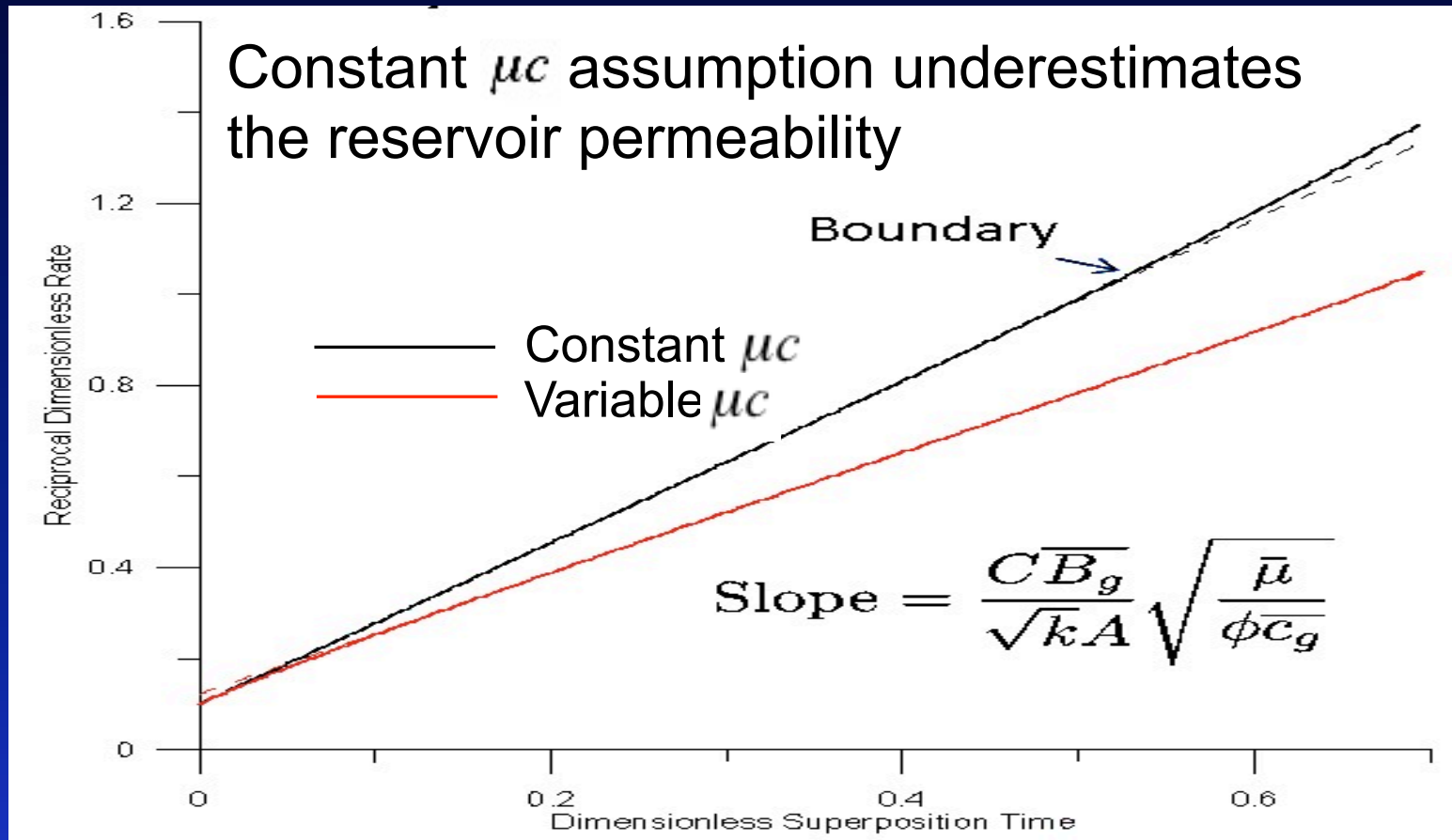
### Effect of Variable Compressibility



# Status

## Phase II

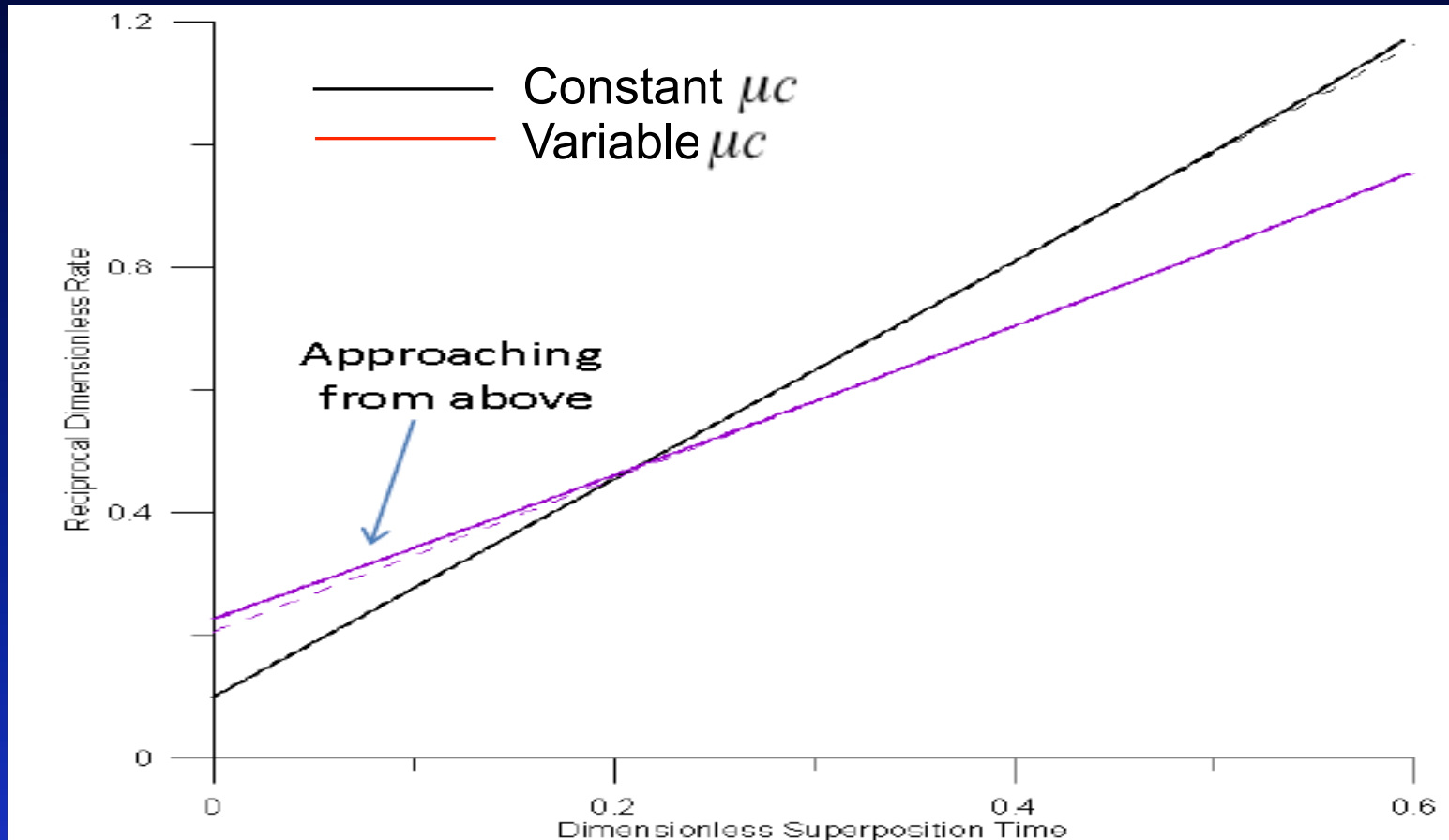
### Superposition Time Plot



# Status

## Phase II

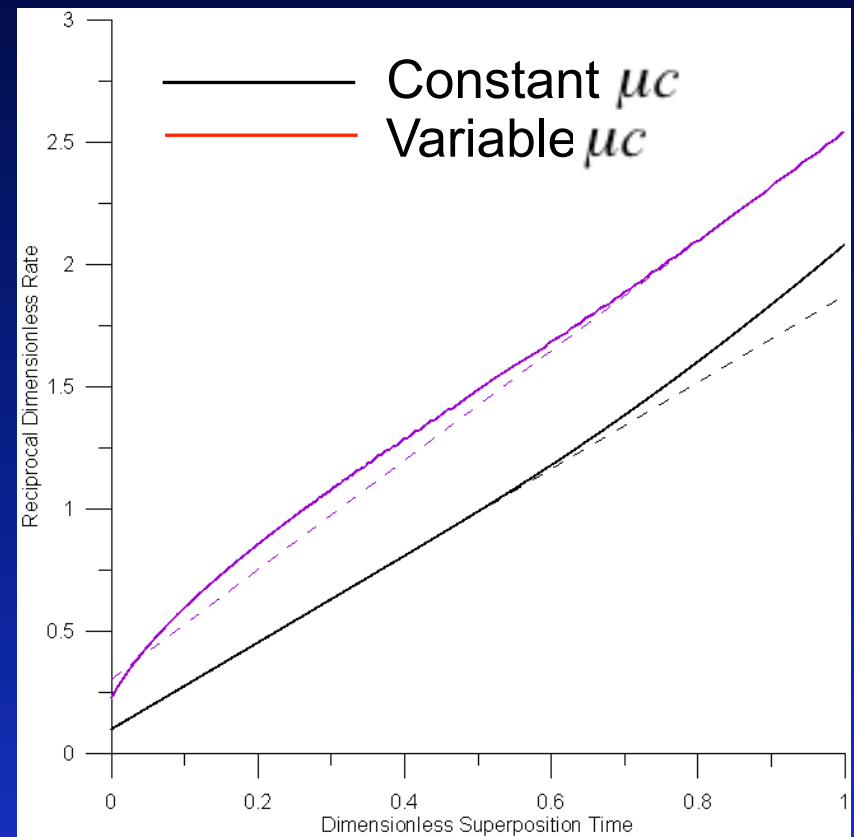
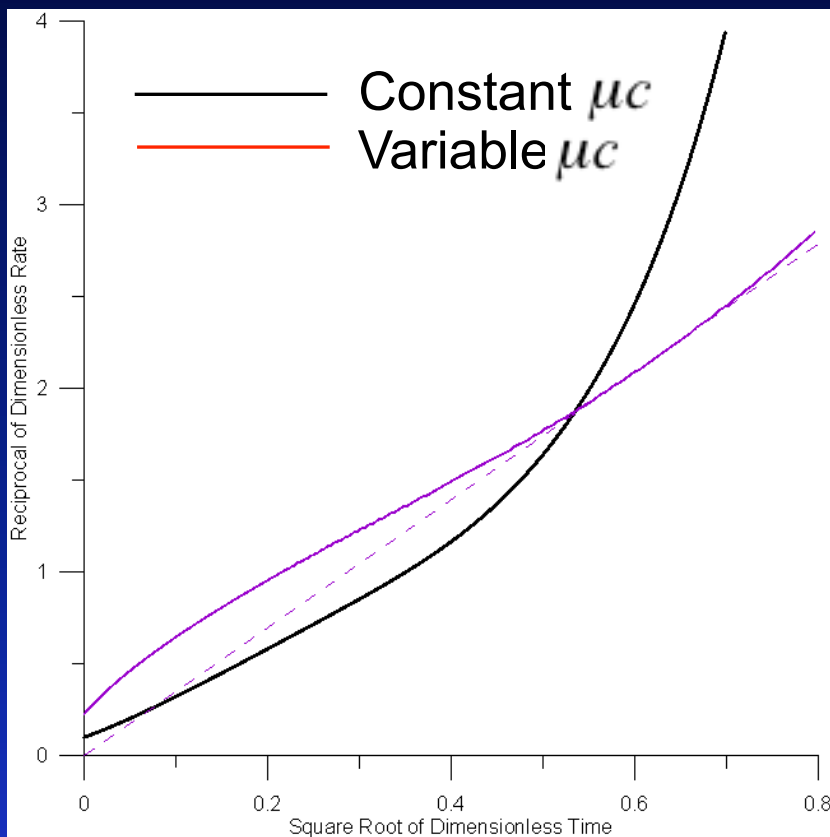
Variable  $c_g \mu_g$  + Non-Darcy effects



# Status

## Phase II

Variable  $c_g \mu_g$  + ND + Stress dependent Permeability





# Work to Continue

## Phase III

Extension of the model need to incorporate more physics

- Pressure dependent permeability
- Multiphase flow effects (water, condensate)
- Klinkenberg effect
- Desorption of gas

