Progress Report

A Spectral Solution for the Analysis of Multifractured Tight-Gas Wells with Large Viscosity-Compressibility Variation and Pressure-Dependent Permeability

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Problem Statement

Analysis of multifractured tight-gas wells

There are two approaches used in the analysis of multifractured tight-gas well performances:

- Analytical modeling
- Numerical simulation

There are known problems with both approaches

The shortcomings of the analysis approaches make the results questionable for most tight-gas well cases

Problem Statement

Analytical Modeling

Continuity equation for flow of real gases in porous media is non-linear.

For analytical modeling, solutions of the continuity equation must be obtained

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{p}{\mu Z}\frac{\partial p}{\partial r}\right)=\frac{\phi}{2.637\times 10^{-4}k}\frac{\partial (p/Z)}{\partial t}$$

Combination of Mass equation, Darcy's Law, and the equation of state

Non-linear

Problem Statement

Classical Approach to the Problem

Real gas pseudopressure is defined to linearize the partial differential equation. According to this common approach;

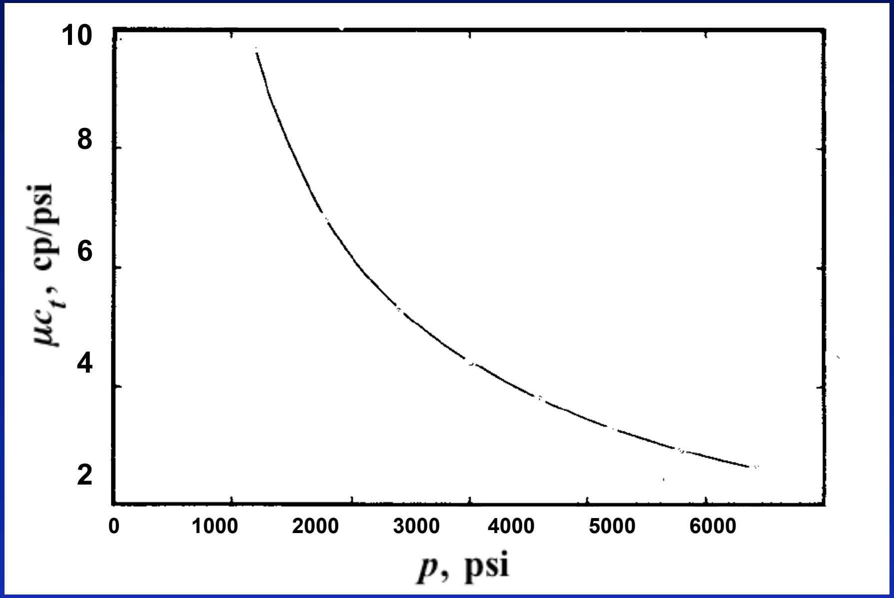
$$m(\mathrm{p}) = 2\int_{p_o}^{p} \frac{p'}{\mu Z}\, dp' \longrightarrow \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial m}{\partial r}\right) = \frac{\phi c_t \mu}{2.637\times 10^{-4} k}\frac{\partial m}{\partial t}$$

Still nonlinear because compressibility and viscosity are functions of pressure

Problem Statement

Analytical Modeling

Pseudopressure linearization assumes that the change in viscosity-compressibility product is negligible



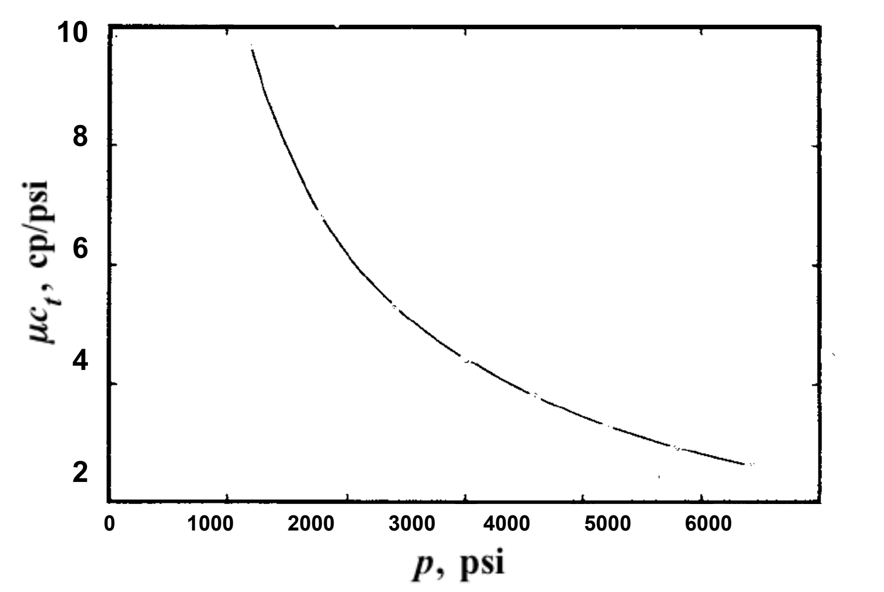
$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial m}{\partial r}\right)=\frac{\phi c_{ti}\mu_i}{2.637\times10^{-4}k}\frac{\partial m}{\partial t}$$

Problem Statement

Analytical Modeling

The assumptions of pseudopressure linearization may be reasonable in the analyses of some conventional reservoirs.

- relatively high pressures
- small pressure changes
- infinite-acting behavior
- pressure-transient analysis
- …
- …



Problem Statement

Analytical Modeling

In tight-gas reservoirs pressure drops are very large; so we have large changes in compressibility and viscosity.

Additional complexities may exist due to

- Pressure dependent permeability
- Non-Darcy flow effects
- Multiphase flow effects
- The Klinkenberg effect

Problem Statement

Numerical Simulation

Current commercial simulators are not able to capture all of the physics, and even if they did, their accuracy would be questionable.

Finite-difference methods

- Approximate the partial differential equations by local algebraic difference equations.
- To obtain accurate solutions, very fine grids and very small time steps need to be used.

Available data usually do not justify the use of detailed numerical simulators.

Objectives

Objectives of the Research

Document the problems encountered and errors incurred in the analysis of tight-gas well performances by pseudopressure linearization

Present an analytical and a semi-analytical spectral solution to analyze the performances of fractured wells in tight-gas reservoirs

Demonstrate the analysis with the new solution and highlight the improvement in results

Scope of Research

Scope of the Research

Investigate the variable compressibility and viscosity product effects on data analysis.

$$C_1 \frac{\partial^2 m(\mathrm{p})}{\partial x^2} = \phi c_g \mu_g \frac{\partial m(\mathrm{p})}{\partial t}$$

Develop a spectral solution of the gas diffusivity equation and incorporate the essential physics of tight-gas reservoirs.

Demonstrate the data analysis with the new solution

Determine if superposition time is applicable and what its limitations are

Spectral Methods

Assumption: The solution of nonlinear diffusion equations is given in the form of an infinite series

Approach 1:

- Approximate infinite series solution as a truncated Chebyshev series

$$p_M(\mathrm{x},\mathrm{t}) \approx \sum_{k=0}^{N} c_k(\mathrm{t})\cos(\mathrm{k}\arccos\mathrm{x})$$

- Substitute the infinite series into the diffusion equation
- Set up the collocation points in space
- Solve for the (time dependent) coefficients

Approach

Spectral Methods

Approach 1 (Continued):

Evaluate the coefficients in the interpolating polynomials for the dependent variables using the collocation method

Require that the governing equations be exactly satisfied at collocation points; we choose these to be the Gauss-Lobatto points

$$x_j = \frac{L_x}{2}\left[1+\cos\left(\frac{j\pi}{N_x}\right)\right], \quad j = 0,1,\ldots,N_x$$

Once the points have been set and the coefficients of the dependent variables have been determined, the polynomial representation of the dependent variables is unique at any time.

Spectral Methods

Approach 2:
Any interpolating polynomial formula for a function $f(x)$ can be equivalently written in terms of Cardinal Functions $C_i(x)$ as

$$f_{N_x}(x) = \sum_{i=0}^{N_x} f(x_i) C_i(x)$$

where $$C_i(x_j) = \delta_{i,j} = \begin{cases} 1 & \text{for } i = j \\ 0 & \text{for } i \neq j \end{cases}$$

and $$C_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{N_x} \frac{x - x_j}{x_i - x_j}$$

Note that the Cardinal function form depends on the x_i form.

"Numerical Recipes – The art of Scientific Computing 3rd Edition", p. 1089

Approach

Spectral Methods

Approach 2 (Continued):
In terms of Cardinal Functions, the pressure function can be written as

$$p(x,t) = \sum_{j=0}^{N_x} p(x_j,t) C_j(x)$$

Differentiate with respect to x

$$\frac{\partial p(x,t)}{\partial x} \approx \sum_{j=0}^{N_x} p(x_j,t) \frac{dC_j(x)}{dx}$$

Spectral Methods

Approach 2 (Continued):
Evaluate at the collocation points

$$\frac{\partial p\left(x_q,t\right)}{\partial x} \approx \sum_{j=0}^{N_x} p\left(x_j,t\right)\frac{dC_j\left(x_q\right)}{dx}$$

In general, we can compute partial derivatives with respect to x at the collocation points via

$$\frac{\partial p\left(x_q,t\right)}{\partial x} \approx D_x p\left(x_q,t\right)$$

Approach

Advantages of Spectral Method

•Derivatives computed exactly and exponential convergence for smooth functions.

•Like the FEM, the spectral method also approximates the solution, but not locally.

•Spectral methods approximate the solution as a linear combination of continuous functions that are generally nonzero throughout the domain(usually sinusoids or Chebychev polynomials)- global approach.

•Nonlinear terms (fluid properties, pressure dependent permeability, non-Darcy flow) are very easy to incorporate.

Disadvantages of Spectral Method

•Spectral Methods work well on simple geometries but harder to implement on complex geometries.

•Time step restrictions for stability using explicit formulations with spectral methods are exorbitant, so implicit formulations are needed.

Approach

Spectral Model – Part 1

Tc, Pc for given composition and specific gravity

1. Sutton1985
2. Sutton2005A
3. Sutton2005C
4. Standing

Z-factor and C_g @P and T

1. Ideal gas
2. Dranchuk - Abu Kassim
3. Hall - Yarborough

Gas viscosity for converged z-factor @P and T

1. Lee Viscosity
2. Lucas Viscosity

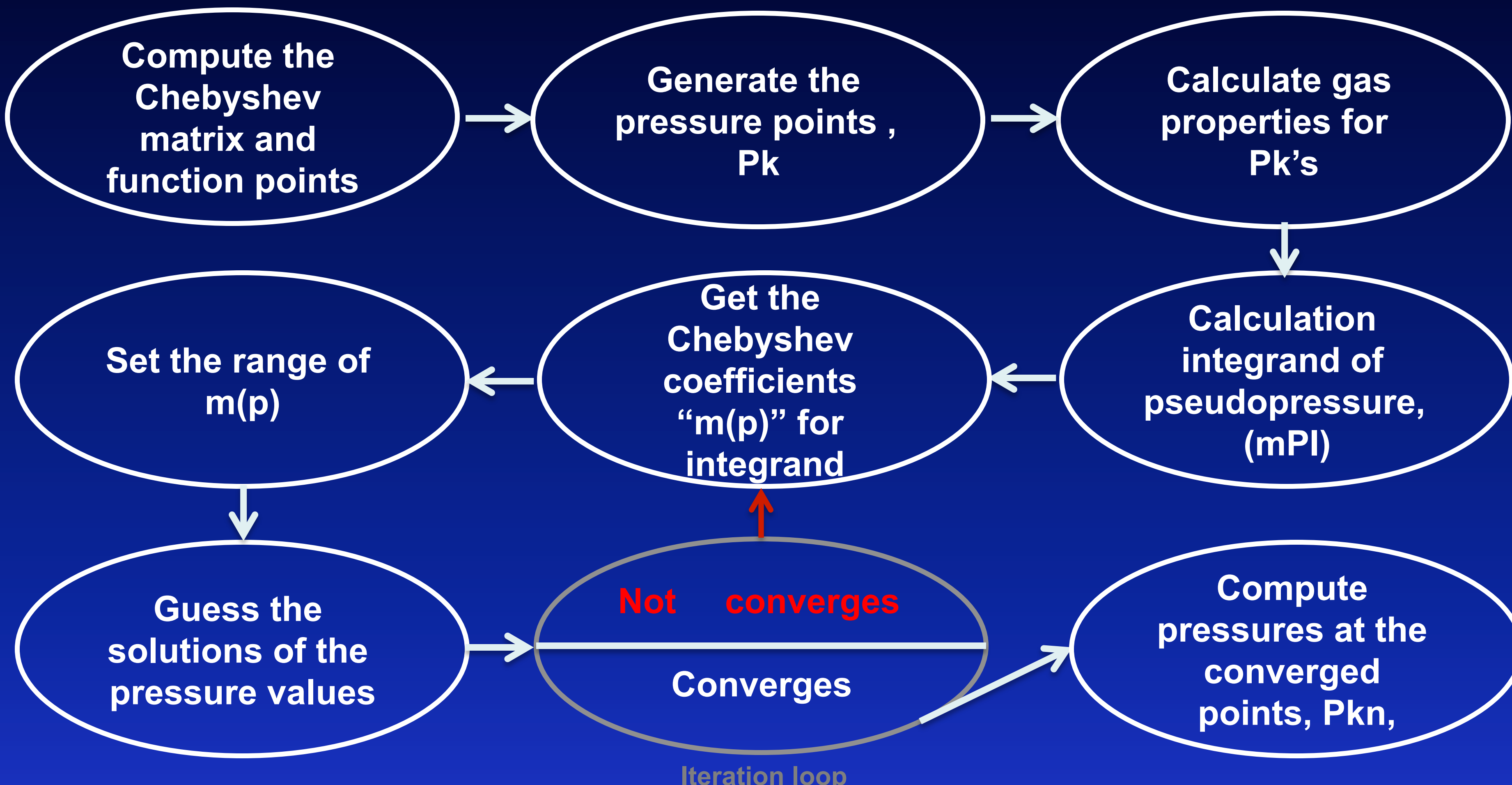
Water Properties "Bw, Rhow, Cw and muw" @P and T

Relative permeability of Gas and Water

1. Corey
2. Modified Corey
3. Generalized Corey
4. X Curves

Approach

Spectral Model – Gas Properties



Current Status of the Research

Base model has been created and ready to implement more detailed physics, such as

- Pressure dependent permeability

- Multiphase flow effects (water, condensate)

- Klinkenberg effect

- Desorption of gas