UNCONVENTIONAL RESERVOIR ENGINEERING PROJECT
COLORADO SCHOOL OF MINES

UREP

CSM

UREP Spring 2017 Advisory Board Meeting

SUPERPOSITION TIME FOR HIGHLY COMPRESSIBLE LINEAR FLOW

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Objectives

- Express 1D flow toward a fractured well in a tight-gas reservoir with strong variability of gas viscosity and compressibility in the form of a perturbation problem .
- Obtain an approximate analytical solution in terms of a series of Green's function solutions to a set of linear problems, which permits term-by-term term application of the superposition principle.
- Derive an approximate superposition time expression for variable rate problems in unconventional gas wells with strong variability of gas viscosity and compressibility

Mathematical Formulation

1D gas flow in porous media

$$\frac{\partial}{\partial y}\left(\frac{p}{z}\frac{k}{\mu}\frac{\partial p}{\partial y}\right) = \frac{\phi c}{2.637\times 10^{-4}}\frac{p}{z}\frac{\partial p}{\partial t}$$

$$m(p) = \int_{p_b}^{p} \frac{2p'}{z\mu} d\,p'$$

$$\frac{\partial^2 \Delta m}{\partial y^2} = \frac{1}{\eta}\frac{\partial \Delta m}{\partial t}$$

$$\eta = \frac{2.637\times 10^{-4} k}{\phi \mu c}$$

Mathematical Formulation

Perturbation problem

$$\frac{\partial^2 \Delta m}{\partial y^2} = (1 + \varepsilon\omega)\frac{1}{\eta_i}\frac{\partial \Delta m}{\partial t}$$

$$\omega = \omega(y,t) = \frac{\eta_i - \eta}{\eta} = \frac{(\phi\mu c)_i - (\phi\mu c)}{(\phi\mu c)}$$

$$\varepsilon = \begin{cases} 0 & \textit{Linear problem} \\ 1 & \textit{Non-Linear problem} \end{cases}$$

$$\eta_i = \frac{2.637\times 10^{-4} k}{(\phi\mu c)_i}$$

Mathematical Formulation

Perturbation problem

$$\frac{\partial^2 \Delta m}{\partial y^2} = (1 + \varepsilon\omega)\frac{1}{\eta_i}\frac{\partial \Delta m}{\partial t}$$

$$\Delta m = \Delta m^0 + \sum_{k=1}^{\infty} \varepsilon^k \Delta m^k$$

Mathematical Formulation

Perturbation problem

$$\left(\frac{\partial^2 \Delta m^0}{\partial y^2} - \frac{1}{\eta_i}\frac{\partial \Delta m^0}{\partial t}\right) + \varepsilon^1\left(\frac{\partial^2 \Delta m^1}{\partial y^2} - \frac{1}{\eta_i}\frac{\partial \Delta m^1}{\partial t} - \frac{\omega^0}{\eta_i}\frac{\partial \Delta m^0}{\partial t}\right) + \varepsilon^2\left(\frac{\partial^2 \Delta m^2}{\partial y^2} - \frac{1}{\eta_i}\frac{\partial \Delta m^2}{\partial t} - \frac{\omega^1}{\eta_i}\frac{\partial \Delta m^1}{\partial t}\right) + \cdots + \varepsilon^k\left(\frac{\partial^2 \Delta m^k}{\partial y^2} - \frac{1}{\eta_i}\frac{\partial \Delta m^k}{\partial t} - \frac{\omega^{k-1}}{\eta_i}\frac{\partial \Delta m^{k-1}}{\partial t}\right) + \cdots = 0$$

Δm^0, Δm^1, Δm^2, …,Δm^k, … are the solutions of 0th, 1st, 2nd, …, kth order perturbation problems:

Mathematical Formulation

Approximate solution:

$$\Delta m(0,t) \approx \frac{1422T\sqrt{\pi\eta_i}}{x_f hk} t_s$$

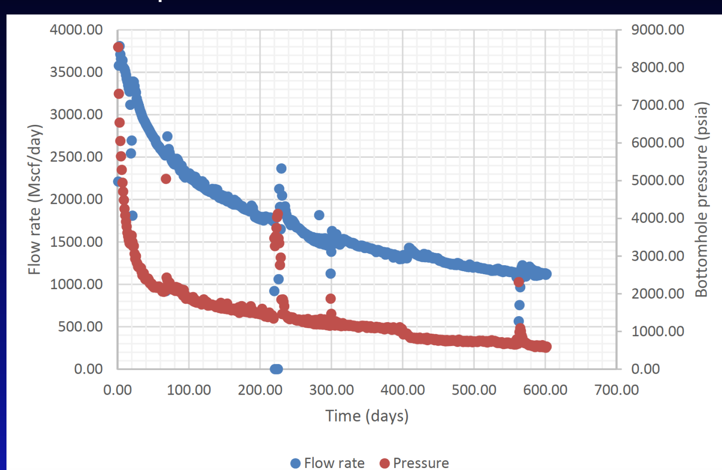
$$t_s = q(\tilde{t}_0)\left[1 - \frac{\omega^0(0,\tilde{t}_0)}{\sqrt{2}}\right]\sqrt{t} + \sum_{j=1}^{M-1}\left[q(\tilde{t}_j) - q(\tilde{t}_{j-1})\right]\left[1 - \frac{\omega^0(0,\tilde{t}_j)}{\sqrt{2}}\right]\sqrt{t - t_j}$$

When the variation of viscosity-compressibility product is negligible

$$t_s = q(\tilde{t}_0)\sqrt{t} + \sum_{j=1}^{M-1}\left[q(\tilde{t}_j) - q(\tilde{t}_{j-1})\right]\sqrt{t - t_j}$$

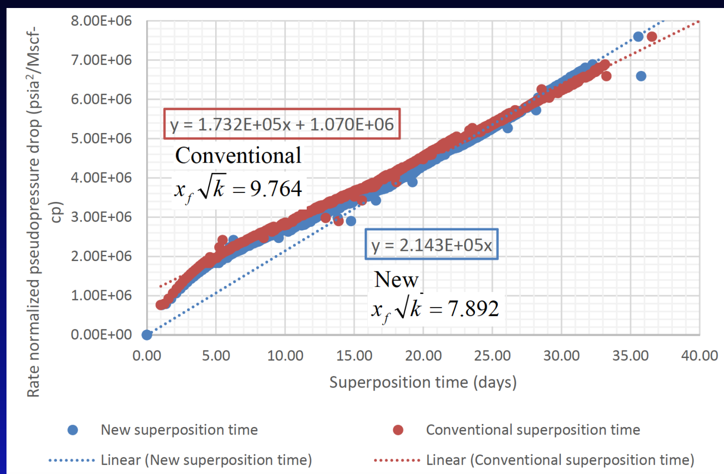
Mathematical Formulation

Field example:



Mathematical Formulation

Field example:



Back-up slides

Mathematical Formulation

Perturbation problem

0th order perturbation problem:

$$\left.\begin{aligned}
&\frac{\partial^2 \Delta m^0}{\partial y^2} - \frac{1}{\eta_i}\frac{\partial \Delta m^0}{\partial t} = 0\\
&\Delta m^0(y, t \to 0) = 0\\
&\Delta m^0(y \to \infty, t) = 0\\
&\left(\frac{\partial \Delta m^0}{\partial y}\right)_{y=0} = -\frac{1422\pi q(t) T}{2x_f hk}
\end{aligned}\right\}$$

1st order perturbation problem:

$$\left.\begin{aligned}
&\frac{\partial^2 \Delta m^1}{\partial y^2} - \frac{1}{\eta_i}\frac{\partial \Delta m^1}{\partial t} - \frac{\omega^0}{\eta_i}\frac{\partial \Delta m^0}{\partial t} = 0\\
&\Delta m^1(y, t \to 0) = 0\\
&\Delta m^1(y \to \infty, t) = 0\\
&\left(\frac{\partial \Delta m^1}{\partial y}\right)_{y=0} = 0
\end{aligned}\right\}$$

Mathematical Formulation

Perturbation problem

2nd order perturbation problem:

$$\left.\begin{aligned}
&\frac{\partial^2 \Delta m^2}{\partial y^2} - \frac{1}{\eta_i}\frac{\partial \Delta m^2}{\partial t} - \frac{\omega^1}{\eta_i}\frac{\partial \Delta m^1}{\partial t} = 0\\
&\Delta m^2(y, t \to 0) = 0\\
&\Delta m^2(y \to \infty, t) = 0\\
&\left(\frac{\partial \Delta m^2}{\partial y}\right)_{y=0} = 0
\end{aligned}\right\}$$

kth order perturbation problem:

$$\left.\begin{aligned}
&\frac{\partial^2 \Delta m^k}{\partial y^2} - \frac{1}{\eta_i}\frac{\partial \Delta m^k}{\partial t} - \frac{\omega^{k-1}}{\eta_i}\frac{\partial \Delta m^{k-1}}{\partial t} = 0\\
&\Delta m^k(y, t \to 0) = 0\\
&\Delta m^k(y \to \infty, t) = 0\\
&\left(\frac{\partial \Delta m^k}{\partial y}\right)_{y=0} = 0
\end{aligned}\right\}$$

Mathematical Formulation

Green's function solutions of the perturbation problem

0th order perturbation problem:

$$\Delta m^0(y,t) = \frac{1422\sqrt{\pi\eta_i}T}{2x_f hk}\int_0^t \frac{q(t')}{\sqrt{t-t'}} exp\left[-\frac{y^2}{4\eta_i(t-t')}\right]dt'$$

1st order perturbation problem:

$$\Delta m^1(y,t) = \frac{1422T}{2x_f hk}\int_0^t q(t')\int_0^\infty \frac{\omega^0(y',t')}{(t-t')} exp\left[-\frac{(y-y')^2+y'^2}{4\eta_i(t-t')}\right]dy'\,dt'$$

Mathematical Formulation

Truncated perturbation solution:

$$\Delta m = \Delta m^0 + \sum_{k=1}^{\infty}\varepsilon^k \Delta m^k \approx \Delta m^0 + \Delta m^1$$

$$= \frac{1422T}{2x_f hk}\left\{\int_0^t \frac{q(t')\sqrt{\pi\eta_i}}{\sqrt{t-t'}}\exp\left[-\frac{y^2}{4\eta_i(t-t')}\right]dt' + \int_0^t q(t')\int_0^\infty \frac{\omega^0(y',t')}{(t-t')} exp\left[-\frac{(y-y')^2+y'^2}{4\eta_i(t-t')}\right]dy'\,dt'\right\}$$

Mathematical Formulation

Discretized truncated perturbation solution:

$$\Delta m(0,t) \approx \frac{1422T\sqrt{\pi\eta_i}}{x_f hk}\sum_{j=0}^{M-1} q(\tilde{t}_j)\left\{\left[1+\frac{\omega^0(\tilde{y}_0',\tilde{t}_j)}{\sqrt{2}}\right]\left[\sqrt{t-t_j}-\sqrt{t-t_{j+1}}\right]\right.$$

$$+\lim_{N\to\infty}\sum_{i=0}^{N}\frac{\left[\omega^0(\tilde{y}_{i+1}',\tilde{t}_j)-\omega^0(\tilde{y}_i',\tilde{t}_j)\right]}{\sqrt{2}}\left\{\left[\sqrt{t-t_j}\,\mathrm{erfc}\left(\frac{y_{i+1}'}{\sqrt{2\eta_i(t-t_j)}}\right)\right.\right.$$

$$\left.-\sqrt{t-t_{j+1}}\,\mathrm{erfc}\left(\frac{y_{i+1}'}{\sqrt{2\eta_i(t-t_{j+1})}}\right)\right]$$

$$\left.\left.+\frac{y_{i+1}'}{\sqrt{2\pi\eta_i}}\left[Ei\left(-\frac{y_{i+1}'^2}{2\eta_i(t-t_j)}\right)-Ei\left(-\frac{y_{i+1}'^2}{2\eta_i(t-t_{j+1})}\right)\right]\right\}\right\}$$